LEBANESE AMERICAN UNIVERSITY

Electrical and Computer Engineering Dept

COE 755

Queueing Theory

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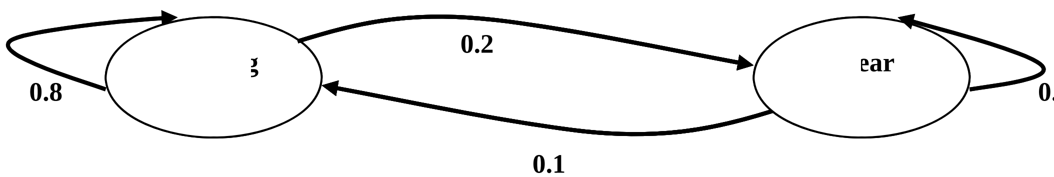
Exam Preparation Problems

I. Markov Chains and Processes

Problem I

The probability of rain tomorrow is 0.8 if it is raining today, and the probability it is clear tomorrow is 0.9 if it is clear today.

- Construct the one-step transition matrix



For the Markov chain in the Figure above, we know that the one-step transition matrix is:

$$P = \begin{pmatrix} 0.8 & 0.2 \\ 0.1 & 0.9 \end{pmatrix}$$

- Find the steady state probabilities.

π_1 = 1/3 and π_2 = 2/3.

Problem II

Consider the following gambler's ruin problem. A gambler bets one unit on each play of a game. He has a probability p = 0.5 of winning and 1-p of losing. He continues to play until he goes broke (state 0) or nets a fortune of 3 units (state 3). (the states 0 and 3 are called *absorbing states*, since the Markov process terminates when it enters one of these two states) Let X_n be the gambler's fortune at the n^{th} play.

- Setup the one-step transition matrix (remember that the system does not exit from states 0 or 3)

$$P = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 1-p & 0 & p & 0 \\ 0 & 1-p & 0 & p \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

- Given that the gambler has 1 unit at time 0, what would be his fortune one step later? In other words, find X_1

X_1 = [0 1 0 0]*P =[0.5 0 0.5 0]=>fortune one step later = 1.

II. Markovian queues

Problem I

A PBX system was installed to handle the voice traffic generated by 300 employees in a factory. If each employee, on the average, makes 1.5 calls per hour with average call duration of 2.5 minutes, what is the offered load presented to this PBX?

Offered load = Arrival rate x service rate = 300 x 1.5 x 2.5 / 60 = 18.75 Erlangs.

Problem II

Variable length data packets arrive at a communication node according to a Poisson process at an average rate of 180 packets per minute. The single outgoing communication link is operating at a transmission rate of 4800 bits per second. The packet lengths can be assumed to be exponentially distributed with an average length of 960 bits.

Calculate the principal performance measures (L, L_q, W, and W_q) of this communication link assuming that it has a very large input buffer.

The average service time = 0.2 s. and the arrival rate λ = 180 packets/min = 3 packets/sec thus ρ=3 x 0.2 = 0.6
The parameters of interest can be calculated easily as follows
$L = \rho/(1-\rho)=3/2$ packets and $L_q = \rho^2/(1-\rho)=9/10$ packets
$W = 1/\mu-\lambda=1/2$ seconds and $W_q = 3/10$ seconds