1. Some Useful Formulas:

1.1. De Morgan's Laws:

$$(A_1 \cup A_2 \dots \cup A_n)^c = A_1^c \cap A_2^c \dots \cap A_n^c$$

$$(A_1 \cap A_2 \dots \cap A_n)^c = A_1^c \cup A_2^c \dots \cup A_n^c$$

1.2. Complements

$$P(A^c) = 1 - P(A)$$

1.3. Unions

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A_1 \cup A_2 \cup \dots \cup A_n) = \sum_{i=1}^{n} P(A_i) \text{, if the } A_i \text{ are disjoint}$$

$$P(A_1 \cup A_2 \cup \dots \cup A_n) \leq \sum_{i=1}^{n} P(A_i)$$

1.4. Intersections

$$P(A \cap B) = P(A)P(B|A) = P(B)P(A|B)$$

1.5. Law of Total Probability (LOTP)

Let E_1, E_2, …, E_n be a partition of the sample space S (i.e., they are disjoint and their union results in all of S). If $P(E_j \neq 0)$ for all j, then

$$P(B) = \sum_{j=1}^{n} P(B|E_j)P(E_j)$$

1.6. Bayes Rule

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

The denominator P(B) is often expanded by the Law of Total Probability.

1.7. Expected Value and Variance

The expected value is linear, that is, for any random variables X and Y and constant c, we have:

$$E(X + Y) = E(X) + E(Y)$$

$$E(cX) = cE(X)$$

The variance of a random variable X is defined as $E[(X-E(X))^2]$, but often it is easier to compute as follows:

$$\boldsymbol{Var(X) = E(X^2) - \left(E(X)\right)^2}$$

Note that $E(X^n)$ represents the n^{th} moment of the random variable X.

In general, it is not true that Var(X+Y) = Var(X) + Var(Y). Constants come out from variance squared:

$$\boldsymbol{Var(cX) = c^2 Var(X)}$$

1.8. Law of the Unconscious Statistician (LOTUS)

Let us consider a discrete random X and let *g* be real-valued function of X. This implies that Y=g(X) is itself a random variable. Normally, to compute E(Y) using the definition of the expected value, we would need first to find the PMF of Y and then go ahead and find E(Y). However, LOTUS states that E(Y) can be found based on the PMF of X as follows:

$$\boldsymbol{E(Y) = E\big(g(X)\big) = \sum_{x} g(x) P(X = x)}$$

Note that the sum is carried out over all possible values of the random variable X. Similarly, for a continuous r.v. X with PDF f_X, we can find the expected value of Y = g(X) by relying on the PDF of X as follows by LOTUS:

$$\boldsymbol{E(Y) = E\big(g(X)\big) = \int_{-\infty}^{\infty} g(x) f_X(x) dx}$$

2. Some Mathematical Facts

2.1. Mapping event terminology to set terminology

Consider a random experiment, where S consists of all the possible outcomes and s ∈ S is an actual outcome of the experiment. Enclosed below is a dictionary mapping between event terminology and set terminology:

Probability	Sets
Sample space	S
A possible outcome s	s ∈ S
Event A	$A \subseteq S$
A or B (inclusive)	$A \cup B$
A and B	$A \cap B$
Not A	A^c
A and B are mutually exclusive	$A \cap B = \phi$
A and B are independent	$P(A \cap B) = P(A)P(B)$

2.2. Partial Derivatives

Partial derivatives are quite analogous to ordinary derivatives. When doing partial derivatives, just treat all the other variables as constants expect the one with respect to which you are differentiating. For example, let $f(x) = y\ \sin(x^2 + y^3)$. Then, the partial derivative with respect to x is given by:

$$\frac{\partial f(x,y)}{\partial x} = 2xy\cos(x^2 + y^3)$$

and the partial derivative with respect to y is:

$$\frac{\partial f(x,y)}{\partial y} = \sin(x^2 + y^3) + 3y^3\cos(x^2 + y^3)$$

2.3. Multiple Integrals

Multiple integrals are analogous to single integrals. In particular, it is a matter of integrating multiple times while holding variables other than the current variable of integration constant. For example,

$$\int_0^1 \int_0^y (x - y)^2\, dx\, dy$$

$$= \int_0^1 \int_0^y (x^2 - 2xy + y^2)\, dx\, dy$$

$$= \int_0^1 \left(\frac{x^3}{3} - x^2 y + xy^2\right)\Bigg|_0^y dy = \int_0^1 (\frac{y^3}{3} - y^3 + y^3)\, dy = \frac{1}{12}$$

2.4. Sums

There are two infinite series results that will pop up over and over again in Queueing Theory, namely the geometric series and the Taylor series of e^x

2.4.1. Geometric series

$$\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}, \text{ for } |x|<1$$

2.4.2. Taylor series for e^x

$$\sum_{n=0}^{\infty} \frac{x^n}{n!} = e^x, \text{ for all } x$$

2.5. Binomial Theorem

The binomial theorem states that

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k}$$

Where $\binom{n}{k}$ is called the binomial coefficient and is defined as the number of ways of choosing k objects from a set of n objects without regard to order. As such, we have

$$\binom{n}{k} = \frac{n!}{(n-k)!\,k!}$$

2.6. A Useful Limit

$$\lim_{n\to\infty} \left(1+\frac{x}{n}\right)^n = e^x \text{, for any real x}$$