Buffer-Aided Relaying Protocols for Cooperative FSO Communications

Chadi Abou-Rjeily, *Senior Member, IEEE*, and Wissam Fawaz, *Senior Member, IEEE*

***Abstract*— In this paper, we consider the problem of relay-assisted free space optical (FSO) communications in the case where the relays are equipped with buffers of finite size. The high directivity of the FSO links clearly distinguishes cooperative FSO networks from their radio frequency (RF) counterparts thus motivating the design of FSO-specific buffer-aided (BA) cooperative protocols. We propose three novel decode-and-forward relaying protocols that are adapted to the nature of FSO transmissions and are capable of achieving different levels of tradeoff between outage probability, average packet delay and system complexity: 1) the BA selective relaying protocol that can be implemented in the presence of channel state information (CSI) and that outperforms the RF *max-link* protocol with a reduced delay; 2) the BA all-active relaying protocol that can be implemented in the absence of CSI and constitutes the simplest protocol with the best delay performance at the expense of a degraded outage performance; and 3) the BA load-balanced selective protocol where supplementary FSO communications are triggered along the inter-relay links for a more balanced distribution of the packets among the buffers. While the last protocol incurs the highest signaling complexity, it results in significant performance gains with a delay that is comparable to that of the BA selective protocol. A Markov chain analysis is adopted for evaluating the system outage probability and the average packet delay where the corresponding state transition matrices are derived in the cases of both symmetrical and asymmetrical networks.**

***Index Terms*— Free-space optics, cooperation, buffers, relay selection, all-active relaying, load balancing.**

I. Introduction

COOPERATIVE communication constitutes an active research area due to its ability to enhance the reliability and extend the coverage of wireless networks while using the existing infrastructure. Cooperative techniques were widely investigated in the context of Free Space Optical (FSO) communication systems as a means of mitigating the limiting effects of the turbulence induced atmospheric scintillation [1]–[7]. The existing research in the area of cooperative FSO communications revolves mainly around all-active and selective parallel-relaying that can be implemented in the absence and presence of channel state information (CSI), respectively. All-active relaying [1]–[5] constitutes a simple and efficient two-slot scheme where the information packet is transmitted from the source (S) to the relays (R's) in the first slot and, subsequently, forwarded from the relays to the destination (D) in the second slot. In this context, no preference is given to any of the relays regardless of the strengths of the underlying source-relay and relay-destination links. On the other hand, selective-relaying privileges the transmission along the strongest end-to-end link thus ensuring enhanced performance levels at the expense of an increased system complexity owing to the need to acquire the full CSI [6], [7]. The strength of the two-hop S-R-D link is dominated by the weakest of its hops and this protocol is referred to as the *max-min* relaying in the open literature on Radio Frequency (RF) wireless communication systems [8]. It has been proven that both all-active and selective relaying extract the full diversity gain and that the superiority of the latter resides in an enhanced coding gain [9]. More recently, inter-relay cooperation has been introduced to further boost the reliability of FSO networks in the case where FSO links are established between the relays [10].

While the existing FSO cooperative schemes [1]–[7], [9], [10] and their RF counterparts [8] assume that the relays have no storage capabilities, more recent research efforts revolved around buffer-aided cooperative systems where buffers (data queues) are introduced at the relay nodes [11]–[16]. In the context of RF systems, it has been proven in the open literature that the deployment of buffers improves both the throughput and diversity gains at the expense of increased packet delays [11]–[16]. To improve the performance of the *max-min* protocol where the same relay is selected for reception and transmission, the RF *max-max* protocol was introduced in [11]. This corresponds to a two-slot protocol where the relay with the best S-R link is selected for reception in the first slot while the relay with the strongest R-D link is selected for transmission in the second slot. The presence of buffers ensures that different relays can be selected for reception and transmission thus reducing the system outage probability. The RF *max-link* protocol was proposed in [12] where communications take place along the strongest link that is selected from all available S-R and R-D links. Leveraging the static two-slot allocation, the *max-link* protocol doubles the achievable diversity gain as compared to the *max-min* and *max-max* protocols. The *max-link* protocol that is based on Decode-and-Forward (DF) cooperation [12] was extended to the context of Amplify-and-Forward (AF) cooperation in [13]. A hybrid buffer-aided RF cooperation

Manuscript received January 28, 2017; revised May 16, 2017, August 2, 2017, and September 29, 2017; accepted September 29, 2017. Date of publication October 6, 2017; date of current version December 8, 2017. The associate editor coordinating the review of this paper and approving it for publication was W. Gerstacker. *(Corresponding author: Chadi Abou-Rjeily.)*

The authors are with the Department of Electrical and Computer Engineering, Lebanese American University, Byblos 961, Lebanon (e-mail: chadi.abourjeily@lau.edu.lb; wissam.fawaz@lau.edu.lb).

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Digital Object Identifier 10.1109/TWC.2017.2759107

scheme that combines the advantages of the *max-max* and *max-link* protocols was proposed and analyzed in [14]. On the other hand, [15] targeted the issue of packet delay and proposed an appropriate relaying scheme where a higher priority is given to the R-D links compared to the S-R links in an attempt to empty the buffers at a higher pace and, hence, minimize the average packet delay. Finally, a relay selection scheme that is based on both the channel state and the buffer state was proposed and analyzed in [16] where it has been proven that this selection results in a smaller average packet delay compared to the *max-link* selection. Finally, it is worth noting that the buffer-aided (BA) relaying techniques are capable of benefiting from both the spatial diversity and time diversity where the information packets are stored until the channel conditions become more favorable. Compared to the time diversity methods that are based on packet interleaving and network coding, the BA relaying techniques do not incur any data-rate loss and any involved joint encoding/decoding, respectively. With the recent advances in the storage technologies, storage capabilities can be incorporated at the communicating nodes with a marginal increase in the cost.

To the authors' best knowledge, despite the extensive literature on BA cooperation in RF systems [11]–[16], this problem has never been considered to date in the context of FSO communications. Some recent work on BA relaying was performed for two-hop single-relay mixed RF and hybrid RF/FSO systems [17]; however, this work is mainly driven by the presence of the RF links and, hence, is not directly related to the scenario that we consider in this paper. For the system considered in [17], a number of mobile users communicate with a relay over RF links in the first hop while, in the second hop, the relay transmits the information to the destination over a hybrid RF/FSO link where a RF link is employed as a backup for the FSO link. Given that the S-R and R-D RF communications occur in the same frequency band, the relay that operates in the half-duplex (HD) mode (with respect to the RF links) needs to adaptively switch between reception and transmission. This resource allocation problem that is imposed by the RF links differs substantially from the pure FSO relaying problem that we consider in this paper where the FSO relays operate in the full-duplex (FD) mode and can simultaneously receive from S and transmit to D. It is important to note that even though the proposed buffer-aided architecture may have a slightly higher cost relative to the existing buffer-free relay-assisted FSO systems, the expected sharp reduction in the pricing of FSO systems as well as buffering would render the studied protocols viable in the next ten years. In point of fact, FSO systems and their associated equipment are getting more and more mature and their costs are thus expected to decrease rapidly with time.

In this paper, we consider the problem of BA relay-assisted FSO communication systems. These systems differ substantially from their RF counterparts making it crucial to propose relaying protocols that are adapted to the nature of FSO transmissions. In fact, while RF transmissions have a broadcast nature, FSO transmissions are highly directional implying that more than one FSO link can be concurrently activated without incurring any interference. Moreover, the FSO relays can smoothly operate in a FD manner since different aligned transceivers are deployed for the sake of establishing the wireless connections with the source and destination nodes. Therefore, unlike the RF-BA-HD cooperative systems [11]–[16] that are restricted by the need to limit transmissions to only one node in each time slot, FSO systems can support multiple simultaneous S-R and R-D transmissions. This substantially alters the system design and offers the capability of introducing FSO-tailored schemes that are appealing in leveraging the excessive delays from which RF BA systems suffer while maintaining advantageous diversity gains.

More specifically, we propose and analyze three novel BA FSO relaying protocols. (i): The BA selective relaying protocol where, in each time slot, the source transmits along a selected S-R link while a selected relay concurrently transmits to D. Both selections are based on the states of the relays' buffers and on the strengths of the underlying FSO channels, thus, necessitating the acquirement of the full CSI. Following from the FD capability at the relays, the same relay can be selected for reception and transmission. (ii): The BA all-active relaying protocol that can be implemented in the absence of CSI where all available S-R and R-D links are simultaneously activated. In this case, the source serves as an orchestrator for the S-R links in order to avoid overloading the buffers with redundant replicas of the same packet. Through an Acknowledgement/Negative-Acknowledgement (ACK/NACK) mechanism between the source and relays, the packet is retained at the relay with the smallest buffer size and dropped from the remaining relays. The concurrent transmissions along the R-D links empty the buffers at a faster pace making this scheme the most advantageous one in terms of the average delay. (iii): The BA load-balanced selective protocol that is inspired form the non-BA inter-relay cooperation scheme [10] and that can be implemented when FSO links are established between the relays. This load balancing approach is intended to complement the BA selective scheme in the case of asymmetrical networks. For such networks, some buffers might be full (resp. empty) most of the time and, hence, the corresponding relays can not be selected for reception (resp. transmission) even if they possess the strongest links thus deteriorating the outage performance. The load balancing scheme attempts to equalize the occupancies of the different buffers by moving the packets from the more congested buffers to the less congested buffers. As in [12], [14], and [15], we analyze the proposed schemes in terms of outage probability and average delay based on the theoretical framework that models the evolution of the relay buffers as a Markov chain. Finally, it is worth noting that while the main strength of buffer-aided solutions (whether in the context of RF or FSO communications) resides in reducing the outage probability, this advantage is associated with a delay rendering such solutions more suitable for delay-tolerant applications. In this context, the subsequent analysis shows that the proposed schemes are capable of achieving different levels of compromise between reliability and delay. While the selective schemes achieve the highest performance levels, the proposed all-active scheme results in very small delays.

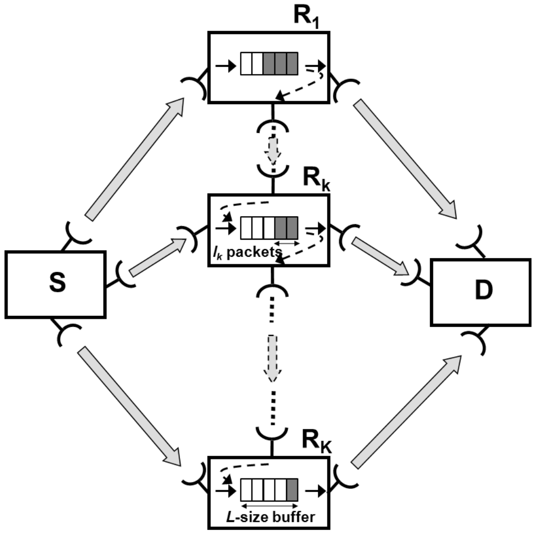

Fig. 1. Buffer-aided cooperative FSO network with K relays and buffers of size L. The dashed links are activated only in the case of inter-relay cooperation (load balancing). In practice, the relays assume arbitrary positions.

In this context, it is worth highlighting that delays can still be experienced by packets in the case of conventional buffer-free relay-assisted FSO systems since a packet that is not correctly received by D would need to be buffered at S for future retransmission. Finally, the presence of the RF backup links in practical systems can leverage the delay requirements where these links can be used to carry the part of the information that is highly delay-sensitive.

II. System Model

A. Basic Parameters

Consider a cooperative FSO network where a source node S communicates with a destination node D through a cluster of K relays denoted by R_1, …, R_K as shown in Fig. 1. We assume that no direct link exists between S and D due, for example, to the large distance separating these nodes or to a blocked line-of-sight link inflicted by the geographical constraints. We also assume that the relays operate in the DF mode where the packets received from S are decoded prior to their retransmission to D. The source node is assumed to have an infinite supply of data in the sense that there is always a packet ready for transmission at each time slot. On the other hand, each relay is equipped with a buffer (data queue) of size L (in number of packets) where the packets received at a certain relay can be temporarily stored if the communication conditions along the corresponding relay-destination link are not favorable. The number of packets in the buffer at the k-th relay is denoted by l_k (where $0 \leq l_k \leq L$) for $k = 1, \dots, K$. Finally, the communication between any two nodes in the network involves the ACK/NACK mechanism where the receiver informs the transmitter about the packet's reception status.

We assume that the packet duration extends over the coherence time of the FSO channel in order to capture the quasi static fading nature of the FSO links. For example, for a coherence time of 1 msec [18] at a data rate of 1 Gbits/s, a buffer of size $L = 1$ corresponds in practice to a memory unit with a storage capability of 0.12 MBytes which falls within the acceptable practical limits. This is especially true since the FSO transceivers are fixed and are much bigger in size compared to the RF mobile nodes.

The relay nodes correspond to independent communication entities that are initially deployed for ensuring wireless optical connectivity between different locations. A natural choice is to install the transceivers in a way to avoid interference with the existing nodes. In case these nodes have no information to communicate, they can serve as relays for assisting S in its communication with D. This constitutes a major advantage of cooperative systems where no additional infrastructure needs to be deployed. In this context, multiple FSO transceivers are present at the destination, each of which is installed for the sake of establishing wireless connectivity with a certain relay as shown in Fig. 1. This holds for practical FSO networks whether user cooperation (BA or non-BA) is implemented or not. In the case where FSO links are preestablished between the relays, these links can be further exploited to improve the system performance. In this paper, we consider the two scenarios of the absence and presence of such inter-relay links. For simplicity, inter-relay links are assumed to exist between two consecutive relays R_{k-1}-R_k for $k = 2, \dots, K$ as shown in Fig. 1. Finally, it is worth noting that the inter-relay links are not deployed for the sake of assisting S in its communication with D, but for the sake of establishing wireless links over which the involved nodes (the relays) can communicate their information. In other words, we are not proposing to add the inter-relay links in case they did not exist, but rather we are proposing a relaying protocol that takes advantage from the potential presence of these links for the sake of achieving enhanced performance levels.

Following from the non-broadcast nature of FSO transmissions, separate FSO transceivers are deployed at each relay for the sake of establishing wireless links with S and D (and, possibly, with the neighboring relays) as shown in Fig. 1. Moreover, the different FSO links do not interfere with each other owing to the high directivity of the laser light beams. These facts overwhelmingly impact the design of cooperative FSO networks where the two following implications arise. (i): The relays can receive from S (or the previous relay) and transmit to D (or the next relay) at the same time and, naturally, the FSO relays operate in the FD mode. (ii): Unlike RF networks, multiple transmissions can occur simultaneously along the S-R, R-D and R-R links which positively impacts the throughput of the network.

For simplicity of notation, the source and destination nodes will be denoted by R_0 and R_{K+1}, respectively. Denote by $h_{i,j}$ the irradiance along the link R_i-R_j. In this work, we adopt the gamma-gamma model where the probability density function (pdf) of the irradiance is given by:

$$f_{i,j}(h) = \frac{2(\alpha_{i,j}\beta_{i,j})^{(\alpha_{i,j}+\beta_{i,j})/2}}{\Gamma(\alpha_{i,j})\Gamma(\beta_{i,j})} h^{(\alpha_{i,j}+\beta_{i,j})/2-1} K_{\alpha_{i,j}-\beta_{i,j}}\left(2\sqrt{\alpha_{i,j}\beta_{i,j}h}\right); \quad h \geq 0, \quad (1)$$

where $\Gamma(.)$ is the Gamma function and $K_c(.)$ is the modified Bessel function of the second kind of order c. The distance-dependent parameters $\alpha_{i,j}$ and $\beta_{i,j}$ are given by: $\alpha_{i,j}^{-1} = \exp\left(0.49\sigma_{R,i,j}^2/(1+1.11\sigma_{R,i,j}^{12/5})^{7/6}\right) - 1$ and $\beta_{i,j}^{-1} = \exp\left(0.51\sigma_{R,i,j}^2/(1+0.69\sigma_{R,i,j}^{12/5})^{5/6}\right) - 1$ where $\sigma_{R,i,j}^2 = 1.23C_n^2 k^{7/6} d_{i,j}^{11/6}$ is the Rytov variance where $d_{i,j}$ stands for the distance between R_i and R_j, k is the wave number and C_n^2 denotes the refractive index structure parameter. Finally, the channel irradiances between the different nodes are assumed to be independent.

We consider a non-coherent FSO system with Intensity-Modulation and Direct-Detection (IM/DD) where the electrical signal-to-noise ratio (SNR) along the link R_i-R_j is given by [1]:

$$\gamma_{i,j} = \frac{\eta^2 G_{i,j}^2 h_{i,j}^2}{N_{\text{link}}^2 N_0}, \tag{2}$$

where η is the optical-to-electrical conversion ratio and N_0 is the variance of the additive white Gaussian noise (AWGN). This noise model constitutes a valid approximation for background noise limited receivers where the shot noise caused by background radiation is dominant with respect to the other noise components such as thermal noise and dark currents [1].

In (2), N_{link} stands for the total number of active links that depends on the implemented cooperation protocol as will be explained later. The normalization by N_{link} ensures that the cooperative system transmits the same power as non-cooperative systems. Finally, $G_{i,j}$ is a gain factor that follows from the fact that the link R_i-R_j might be shorter than the direct link S-D and, hence, will benefit from a higher SNR. This distance-dependent gain factor is given by [1]:

$$G_{i,j} = \left(\frac{d_{0,K+1}}{d_{i,j}}\right)^2 e^{-\sigma(d_{i,j}-d_{0,K+1})}, \tag{3}$$

where σ is the attenuation coefficient.

The link R_i-R_j is said to be in outage if the SNR along this link falls below a certain threshold SNR γ_{th} that ensures the correct decodability of the received packet [1]. From (2), the outage probability can be written as:

$$\begin{aligned} p_{i,j} &= \Pr\left(\gamma_{i,j} < \gamma_{th}\right) \\ &= \Pr\left(h_{i,j} < \frac{N_{\text{link}}}{G_{i,j} P_M}\right) = F_{i,j}\left(\frac{N_{\text{link}}}{G_{i,j} P_M}\right), \end{aligned} \tag{4}$$

where $P_M \triangleq \frac{\eta}{\sqrt{\gamma_{th} N_0}}$ denotes the optical power margin of the average SNR with respect to the threshold SNR γ_{th}. In (4), $F_{i,j}(.)$ corresponds to the cumulative distribution function (cdf) of the gamma-gamma distribution defined in (1). From [6], this cdf is given by:

$$F_{i,j}(h) = \frac{1}{\Gamma(\alpha_{i,j})\Gamma(\beta_{i,j})} G_{1,3}^{2,1}\left[\alpha_{i,j}\beta_{i,j} h \middle|_{\alpha_{i,j},\beta_{i,j},0}^{1}\right]; \quad h \geq 0, \tag{5}$$

where $G_{p,q}^{m,n}[.]$ is the Meijer G-function.

Finally, it is worth noting that adding buffers to the relays does not render the cooperative system capable of mitigating severe weather conditions like fog. In such scenarios, the attenuation can reach several hundreds of dBs rendering all forms of infrared FSO light communications impossible whether with the existing non-buffer-aided relaying schemes [1]–[7] or with the proposed buffer-aided schemes. In this context, all cooperative diversity methods (whether BA or non-BA) are designed not to combat the long-term attenuation but rather to combat the shorter-term scintillation phenomenon. The only remedy to the above situations resides in using alternative communication channels that are not affected by the corresponding weather conditions such as the RF channels where many recent contributions tackled the problem of hybrid RF/FSO systems. In this context, under extreme weather conditions, the considered BA cooperative network can switch to the RF mode in a way that is completely analogous to the non-cooperative and non-BA cooperative networks. In this case, while any of the existing RF buffer-aided schemes [11]–[16] can be readily applied in our system when the low-speed RF mode is activated (the FSO links are down), the proposed FSO schemes result in better advantages under less extreme weather conditions where the FSO links are not completely opaque.

B. Definitions

A source-relay link is considered to be available if the buffer at the relay is not full so that this relay can receive a packet from S. It is worthwhile noting in this regard that no packets are transmitted from the source to a relay for as long as the relay's buffer is full and that packet transmission to the relay resumes once some spare room is created at the relay's buffer through R-D packet transmissions. In the case where all S-R links are in outage and/or all relays' buffers are full, the packet will be stored at the source's buffer. However, this scenario occurs with a very low probability in the average-to-large SNR range; the range in which the relay-assisted fading-mitigation techniques are typically designed to operate and achieve the desirable performance gains. Similarly, a relay-destination link is considered to be available if the buffer at the relay is not empty so that a packet can be forwarded to D. Consequently, the sets $\mathcal{C}_r$ and $\mathcal{C}_t$ of the relays that are available for reception and transmission, respectively, can be expressed as:

$$\mathcal{C}_r \triangleq \{k = 1, \ldots, K | l_k \neq L\}; \quad |\mathcal{C}_r| \triangleq \phi, \tag{6}$$

$$\mathcal{C}_t \triangleq \{k = 1, \ldots, K | l_k \neq 0\}; \quad |\mathcal{C}_t| \triangleq \psi. \tag{7}$$

We also define the set $\mathcal{C}_{r,t}$ of relays that can receive and transmit as:

$$\mathcal{C}_{r,t} \triangleq \mathcal{C}_r \cap \mathcal{C}_t; \quad |\mathcal{C}_{r,t}| \triangleq \theta. \tag{8}$$

In what follows, the strength of the link R_i-R_j will be captured by the random variable $G_{i,j}h_{i,j}$. From (4), the probability that the best available S-R link is in outage can be calculated as follows:

$$\begin{aligned} P_{\mathcal{C}_r} &\triangleq \Pr\left(\max_{k\in\mathcal{C}_r}\{G_{0,k}h_{0,k}\} < \frac{N_{\text{link}}}{P_M}\right) \\ &= \prod_{k\in\mathcal{C}_r} \Pr\left(h_{0,k} < \frac{N_{\text{link}}}{G_{0,k}P_M}\right) = \prod_{k\in\mathcal{C}_r} p_{0,k}, \end{aligned} \tag{9}$$

which is the same as the probability that all available S-R links are in outage.

Similarly, the probability that the best available R-D link is in outage can be written as:

$$Q_{\mathcal{C}_t} \triangleq \prod_{k\in\mathcal{C}_t} p_{k,K+1}, \tag{10}$$

which is the same as the probability that all available R-D links are in outage.

In what follows, a network is defined as symmetrical if all relays are at the same distance from the source and at the same distance from the destination. In other words, $d_{0,1} = \cdots = d_{0,K}$ and $d_{1,K+1} = \cdots = d_{K,K+1}$. In this case:

$$p_{0,1} = \cdots = p_{0,K} \triangleq p; \quad p_{1,K+1} = \cdots = p_{K,K+1} \triangleq q, \tag{11}$$

implying, from (9) and (10), that:

$$P_{\mathcal{C}_r} = p^{\phi}; \quad P_{\mathcal{C}_t} = q^{\psi}. \tag{12}$$

For asymmetrical networks, the relays are numbered in an ascending order according to their distances from the source: $d_{0,1} \leq \cdots \leq d_{0,K}$.

C. State Transition Matrix

For all proposed schemes, a Markov chain analysis is adopted as the theoretical framework for analyzing the evolution of the K buffers. A state represents the numbers of packets present in each buffer and is defined by $(l_1, \ldots, l_K)$ resulting in $(L+1)^K$ possible states. The state transition matrix captures the evolution between the states and comprises the probabilities of going from one state to another. The state transition matrix will be denoted by $\mathbf{A}$ that corresponds to a $(L+1)^K \times (L+1)^K$ matrix whose (i, j)-th element is defined as:

$$\begin{aligned}\mathbf{A}_{i,j} &= \Pr\left((l_1, \ldots, l_K) \to (l'_1, \ldots, l'_K)\right);\\ i &= \mathfrak{N}[(l'_1, \ldots, l'_K)], \quad j = \mathfrak{N}[(l_1, \ldots, l_K)],\end{aligned} \tag{13}$$

where the function $j = \mathfrak{N}[(l_1, \ldots, l_K)] = 1 + \sum_{k=1}^{K} l_k(L+1)^{K-k}$ is used to number the states and defines a one-to-one relation between the set of all possible states $\{0, \ldots, L\}^K$ and the set of integers $\{1, \ldots, (L+1)^K\}$. The inverse relation will be denoted by $\mathfrak{N}^{-1}[j]$ in what follows. The evaluation of the state transition matrix is central for deriving the system outage probability and average packet delay as will be highlighted in Section VI.

III. Buffer-Aided Selective Relaying

A. Cooperation Strategy

We first consider the case where the inter-relay links do not exist and propose a Selective-Relaying (SR) protocol that can be implemented in the case where full CSI is available. For the proposed SR scheme, transmissions take place concurrently along the strongest available S-R and R-D links. In other words, in each time slot, two nodes in the network may be simultaneously transmitting; namely, the source and a selected relay. Therefore, the total power needs to be split among these two links and $N_{\text{link}} = 2$ in (2). Evidently, the selection of the strongest links requires the estimation of the $2K$ S-R and R-D path gains. The selection of the best S-R link is performed by S while the selection of the best R-D link can be orchestrated by D.

The FSO SR protocol corresponds to the selection of the links S-R$_{\hat{k}_r}$ and R$_{\hat{k}_t}$-D where:

$$\hat{k}_r = \arg\max_{k\in\mathcal{C}_r}\{G_{0,k}h_{0,k}\}; \quad \hat{k}_t = \arg\max_{k\in\mathcal{C}_t}\{G_{k,K+1}h_{k,K+1}\}, \tag{14}$$

where the sets $\mathcal{C}_r$ and $\mathcal{C}_t$ are defined in (6)–(7).

The non-buffer-aided equivalent to the considered SR protocol is the *max-min* selective scheme proposed in [6] where transmissions take place along the strongest link S-R$_{\hat{k}}$-D with $\hat{k} \triangleq \arg\max_{k\in\{1,\ldots,K\}}\{\min\{G_{0,k}h_{0,k}, G_{k,K+1}h_{k,K+1}\}\}$. In both cases, the selection involves the knowledge of $\{G_{0,k}h_{0,k}, G_{k,K+1}h_{k,K+1}\}_{k=1}^{K}$. It is worth noting that unlike the *max-min* buffer-free selection scheme that involves the selection of the best end-to-end S-R-D link, the presence of buffers at the relays implies that different relays might be selected for reception and transmission. In other words, the integers $\hat{k}_r$ and $\hat{k}_t$ can be selected independently in (14). The independent selection of the best S-R and R-D links implies that no feedback is needed between D and S.

The proposed SR protocol can be considered as an extension and adaptation of the *max-max* protocol [11] to the context of FSO systems. (i): The adaptation follows since in the RF *max-max* protocol, the time is slotted into two slots where link S-R$_{\hat{k}_r}$ is activated in the first slot while the link R$_{\hat{k}_t}$-D is activated in the second slot. As has been highlighted above, the FSO links are very directive and do not interfere with each other and, consequently, the two-slot scheduling is not required in the case of FSO justifying the concurrent activation of two links. (ii): The extension follows since the analysis in [11] is based on the assumption that no buffers can be full or empty and, thus, the selection is carried out among all relays (rather than the sets $\mathcal{C}_r$ and $\mathcal{C}_t$). On the other hand, the *max-link* protocol in [12] involves the transmission along the single link S-R$_{\hat{k}_r}$ if $G_{0,\hat{k}_r}h_{0,\hat{k}_r} > G_{\hat{k}_t,K+1}h_{\hat{k}_t,K+1}$ and along the link R$_{\hat{k}_t}$-D otherwise, thus, highlighting the difference with the proposed scheme. Finally, while $N_{\text{link}} = 1$ for [11], [12], $N_{\text{link}} = 2$ for the proposed SR FSO scheme. Moreover, the Markov chain analysis of the proposed scheme differs substantially from [12] since the FSO relays operate in the FD mode rather than the HD mode.

B. State Transition Matrix

1) Probability Definitions: We first define the probability $S_{\mathcal{C}_r,i}$ as the probability that the link S-R$_i$ has the maximum strength among the links $\{\text{S-R}_k\}_{k\in\mathcal{C}_r}$ and, hence, the source will transmit along this link. This probability can be evaluated as follows:

$$S_{\mathcal{C}_r,i} \triangleq \Pr\left(G_{0,i}h_{0,i} > \max_{k\in\mathcal{C}_r\setminus\{i\}}\{G_{0,k}h_{0,k}\}\right). \tag{15}$$

Defining the random variable H as $H = \max_{k\in\mathcal{C}_r\setminus\{i\}}\left\{\frac{G_{0,k}}{G_{0,i}}h_{0,k}\right\}$, (15) can be written as $\Pr\left(h_{0,i} > H\right) =$

$\int_0^{+\infty} f_{0,i}(h) \int_0^h f_H(h')\mathrm{d}h'\mathrm{d}h = \int_0^{+\infty} f_{0,i}(h) F_H(h)\mathrm{d}h$ since all involved random variable are positive where $f_{0,i}(.)$ is given in (1) while $f_H(.)$ and $F_H(.)$ stand for the pdf and cdf of H, respectively. Now, $F_H(h) = \Pr(H < h) = \prod_{k\in\mathcal{C}_r\setminus\{i\}} \Pr\left(\frac{G_{0,k}}{G_{0,i}} h_{0,k} < h\right) = \prod_{k\in\mathcal{C}_r\setminus\{i\}} F_{0,k}\left(\frac{G_{0,i}}{G_{0,k}} h\right)$ where $F_{0,k}(.)$ is given in (5). Therefore, (15) simplifies to:

$$S_{\mathcal{C}_r,i} = \int_0^{+\infty} f_{0,i}(h) \prod_{k\in\mathcal{C}_r\setminus\{i\}} F_{0,k}\left(\frac{G_{0,i}}{G_{0,k}} h\right)\mathrm{d}h. \quad (16)$$

In a similar way, we define the probability $S_{\mathcal{C}_t,j}$ as the probability that the link R$_j$-D has the maximum strength among the links $\{\text{R}_k\text{-D}\}_{k\in\mathcal{C}_t}$ and, hence, the relay R$_j$ will be selected to transmit to D. Following the same procedures as above, this probability can be calculated from:

$$S_{\mathcal{C}_t,j} = \int_0^{+\infty} f_{j,K+1}(h) \prod_{k\in\mathcal{C}_t\setminus\{j\}} F_{k,K+1}\left(\frac{G_{j,K+1}}{G_{k,K+1}} h\right)\mathrm{d}h. \quad (17)$$

Given the involved pdf and cdf expressions in (1) and (5), the integrals in (16) and (17) need to be evaluated numerically.

2) Transition Probabilities (General Case): In what follows, we define $\mathbf{e}_i$ as the i-th row of the $K \times K$ identity matrix. For the SR scheme, four types of transitions are possible as follows.

Type-I $(l_1,\ldots,l_K) \to (l_1,\ldots,l_K)$: Assume that the links S-R$_i$ ($i \in \mathcal{C}_r$) and R$_j$-D ($j \in \mathcal{C}_t$) are selected. If $i \neq j$, the only possibility for the buffers to keep the same sizes is when both selected links are in outage which occurs with probability $P_{\mathcal{C}_r} Q_{\mathcal{C}_t}$. On the other hand, if $i = j$, then the same relay is selected for reception and transmission. In this case, the buffers will keep the same sizes either when both links are in outage (no packets are received or transmitted) or when both links are not in outage (one packet is received and one packet is transmitted) implying that the corresponding probability will be $P_{\mathcal{C}_r} Q_{\mathcal{C}_t} + (1-P_{\mathcal{C}_r})(1-Q_{\mathcal{C}_t})$. As a conclusion, the probability of a transition of Type-I (diagonal elements of $\mathbf{A}$) can be written as:

$$p^{(I)} = \sum_{i\in\mathcal{C}_r}\sum_{j\in\mathcal{C}_t} S_{\mathcal{C}_r,i} S_{\mathcal{C}_t,j}\left[P_{\mathcal{C}_r} Q_{\mathcal{C}_t} + \delta_{i,j}(1-P_{\mathcal{C}_r})(1-Q_{\mathcal{C}_t})\right], \quad (18)$$

where $\delta_{i,j}$ stands for the Kronecker delta function ($\delta_{i,j} = 0$ if $i \neq j$ and $\delta_{i,j} = 1$ if $i = j$) while the probabilities $P_{\mathcal{C}_r}$, $Q_{\mathcal{C}_t}$, $S_{\mathcal{C}_r,i}$ and $S_{\mathcal{C}_t,j}$ are defined in (9), (10), (16) and (17), respectively.

Type-II $(l_1,\ldots,l_K) \to (l_1,\ldots,l_K) + \mathbf{e}_i$ ($i \in \mathcal{C}_r$): In this case, the size of the buffer at R$_i$ increases by 1 implying that this relay has been selected for reception and that the packet has been successfully received with no outage. On the other hand, the concurrently selected R-D link is in outage otherwise the size of the buffer at a ceratin relay R$_j$ will drop by 1 if $j \neq i$ or go to 0 if $j = i$. Therefore, a Type-II transition probability is given by:

$$p^{(II)} = S_{\mathcal{C}_r,i}(1 - P_{\mathcal{C}_r}) Q_{\mathcal{C}_t}. \quad (19)$$

Type-III $(l_1,\ldots,l_K) \to (l_1,\ldots,l_K) - \mathbf{e}_j$ ($j \in \mathcal{C}_t$): In this case, the size of the buffer at R$_j$ decreases by 1 implying that this relay has been selected for transmission and that the transmission was successful. On the other hand, the S-R hop of the network should be in outage since no increase in any buffer size was obtained. Consequently:

$$p^{(III)} = S_{\mathcal{C}_t,j}(1 - Q_{\mathcal{C}_t}) P_{\mathcal{C}_r}. \quad (20)$$

Type-IV $(l_1,\ldots,l_K) \to (l_1,\ldots,l_K) + \mathbf{e}_i - \mathbf{e}_j$ ($i \in \mathcal{C}_r$; $j \in \mathcal{C}_t$; $i \neq j$): In this case, relays R$_i$ and R$_j$ are selected for reception and transmission, respectively, implying that:

$$p^{(IV)} = S_{\mathcal{C}_r,i} S_{\mathcal{C}_t,j}(1 - P_{\mathcal{C}_r})(1 - Q_{\mathcal{C}_t}). \quad (21)$$

3) Transition Probabilities (Symmetrical Networks): In this case, the S-R links are identically distributed resulting in $S_{\mathcal{C}_r,i} = \frac{1}{\phi}\ \forall\ i \in \mathcal{C}_r$ from (16) where each one of these links can be selected with the same probability. In the same way, the R-D links are identically distributed and $S_{\mathcal{C}_t,j} = \frac{1}{\psi}\ \forall\ j \in \mathcal{C}_t$. Replacing these values as well as (12) in (18) results in:

$$\begin{aligned} p^{(I)} &= \frac{\phi\psi - \theta}{\phi\psi} p^{\phi} q^{\psi} + \frac{\theta}{\phi\psi}\left[p^{\phi} q^{\psi} + (1 - p^{\phi})(1 - q^{\psi})\right] \\ &= p^{\phi} q^{\psi} + \frac{\theta}{\phi\psi}(1 - p^{\phi})(1 - q^{\psi}), \end{aligned} \quad (22)$$

where θ is defined in (8).

In a similar way, (19), (20) and (21) can be written as: $p^{(II)} = \frac{1}{\phi}(1 - p^{\phi}) q^{\psi}$, $p^{(III)} = \frac{1}{\psi}(1 - q^{\psi}) p^{\phi}$ and $p^{(IV)} = \frac{1}{\phi\psi}(1 - p^{\phi})(1 - q^{\psi})$, respectively.

IV. Buffer-Aided All-Active Relaying

A. Cooperation Strategy

In order to bypass the channel estimation that might be challenging especially for large numbers of relays, we next propose a buffer-aided All-active-Relaying (AR) protocol that can be implemented in the absence of CSI. On the other hand, the transmit power is evenly split among the $2K$ S-R and R-D links resulting in $N_{\text{link}} = 2K$.

For AR, the source transmits in a non-selective manner to all relays. In this case, all relays whose S-R links are not in outage and whose buffers are not full will be able to receive and store the transmitted packet. In the same way, all relays with non-empty buffers are allowed to transmit in a concurrent way to D. While this strategy can be accomplished in a simple manner, the protocol needs to be improved in order to avoid flooding the network's links and the relays' buffers with redundant replicas of the same packet. In fact, since S is transmitting to all relays, then multiple relays might successfully decode the packet and store it in their corresponding buffers. Moreover, concerning the R-D hop, a number of relays might still attempt to transmit a replica of a packet that has been previously delivered to D by a different relay.

In order to alleviate the above problem, the implemented ACK/NACK mechanism needs to be complemented as follows. In the first S-R hop, the relay R$_k$ with successful detection will not reply with an ACK but rather with a short signaling packet of size $\log_2(l_k)$ bits indicating the current number of packets in its buffer. Now, the source will reply by a 1-bit message (along each S-R link) informing the relay with minimum occupancy to keep the packet in its buffer and informing the remaining relays to drop this packet. This procedure will solve the problem of packet replication while balancing out the numbers of packets in the K buffers. In the second R-D hop, all relays with non-empty buffers proceed with the transmission to D as before without any alteration of the ACK/NACK mechanism.

The additional overhead resulting from the proposed AR protocol is judged to be nominal where the size of the additional $\log_2(l_k)$-bit and 1-bit signaling packets is small compared to the size of the information packets since the buffer sizes are not very large. This is especially true given the large coherence time of the FSO channels implying that packets of big sizes can be used. In all circumstances, the AR signaling procedure is much simpler than accomplishing a perfect estimation of the channel gains especially in the presence of excessive noise.

Finally, it is worth noting that when ties occur, the relay with the highest index (i.e. the farthest from S based on the adopted notation) will be selected to keep the packet. This will also contribute to the load balancing since the buffers of the relays that are closer to the source fill up at a higher rate since the SNR along the corresponding S-R link is higher.

B. State Transition Matrix

1) Definitions: We define $\mathcal{H}_{\mathcal{S},i}$ as the subset of the set of relay indices $\mathcal{S}$ that have a higher priority for reception than relay R_i (i.e. smaller buffer size or same buffer size with a higher index). In other words, if a packet is successfully received at R_i and R_j for $j \in \mathcal{H}_{\mathcal{S},i}$, then the relay will be dropped at R_i. This set can be written as follows:

$$\mathcal{H}_{\mathcal{S},i} = \{k \in \mathcal{S} \mid l_k < l_i\} \cup \{k \in \mathcal{S},\ k > i \mid l_k = l_i\}, \tag{23}$$

where $\mathcal{S} \subset \{1, \dots, K\}$ and where the second set follows from the tie breaking rule.

Following from the dropping strategy adopted in the first hop, the probability that R_i (among the relays in $\mathcal{S}$) does not drop a received packet is:

$$(1 - p_{0,i}) \prod_{i' \in \mathcal{H}_{\mathcal{S},i}} p_{0,i'}, \tag{24}$$

where the first term follows since S-R_i should not be in outage for the successful reception of the packet at R_i. The second term follows since the relays with higher reception priority should suffer from outage; otherwise, the packet will be kept at one of these relays rather than R_i.

2) Transition Probabilities: Based on the proposed AR strategy, the number of packets in only one buffer (at most) can increase by 1 while the numbers of packets in any number of buffers can drop by 1. Consequently, four types of transitions are possible as follows.

Type-I $(l_1, \dots, l_K) \rightarrow (l_1, \dots, l_K)$: In Appendix A, we prove that the probability of a Type-I transition is given by:

$$p^{(I)} = \prod_{k \in \mathcal{C}_r \setminus \mathcal{C}_t} p_{0,k} \prod_{k' \in \mathcal{C}_t \setminus \mathcal{C}_r} p_{k',K+1} \times \Bigg[\prod_{i \in \mathcal{C}_{r,t}} p_{0,i}\, p_{i,K+1} + \sum_{i \in \mathcal{C}_{r,t}} (1 - p_{0,i}) \times \prod_{i' \in \mathcal{H}_{\mathcal{C}_{r,t},i}} p_{0,i'} (1 - p_{i,K+1}) \prod_{j \in \mathcal{C}_{r,t} \setminus \{i\}} p_{j,K+1} \Bigg], \tag{25}$$

that simplifies to the following expression in the case of symmetrical networks:

$$p^{(I)} = p^{\phi-\theta} q^{\psi-\theta} \left[p^\theta q^\theta + (1 - p^\theta)(1 - q) q^{\theta-1} \right]. \tag{26}$$

Type-II $(l_1, \dots, l_K) \rightarrow (l_1, \dots, l_K) + \mathbf{e}_i$ $(i \in \mathcal{C}_r)$: In this case, the transition probability can be written as:

$$p^{(II)} = (1 - p_{0,i}) \prod_{i' \in \mathcal{H}_{\mathcal{C}_r,i}} p_{0,i'} \prod_{j \in \mathcal{C}_t} p_{j,K+1}. \tag{27}$$

where the proof is provided in Appendix B. In the case of symmetrical networks, (27) can be written as $p^{(II)} = (1 - p) p^\xi q^\psi$ where $\xi \triangleq |\mathcal{H}_{\mathcal{C}_r,i}|$.

Type-III $(l_1, \dots, l_K) \rightarrow (l_1, \dots, l_K) + \mathbf{e}_i - \sum_{j \in \mathcal{S}} \mathbf{e}_j$ $(i \in \mathcal{C}_r\ ;\ \mathcal{S} \subset \mathcal{C}_t\ ;\ i \notin \mathcal{S})$: In this case, the transition probability can be written as:

$$p^{(III)} = (1 - p_{0,i}) \prod_{i' \in \mathcal{H}_{\mathcal{C}_r,i}} p_{0,i'} \prod_{j \in \mathcal{S}} (1 - p_{j,K+1}) \prod_{j' \in \mathcal{C}_t \setminus \mathcal{S}} p_{j',K+1}, \tag{28}$$

where the first two terms correspond to (24) and follow from storing the packet in the buffer of R_i. The third term corresponds to the probability of successful retransmissions from relays in $\mathcal{S}$ (the corresponding R-D links are not in outage) while the fourth term corresponds to the probability that the remaining R-D links are in outage.

In the symmetrical case, (28) simplifies to $p^{(III)} = (1 - p) p^\xi (1 - q)^{|\mathcal{S}|} q^{\psi - |\mathcal{S}|}$.

Type-IV $(l_1, \dots, l_K) \rightarrow (l_1, \dots, l_K) - \sum_{j \in \mathcal{S}} \mathbf{e}_j$ $(\mathcal{S} \subset \mathcal{C}_t)$: In Appendix C, we prove that the probability of a Type-IV transition is given by:

$$p^{(IV)} = \prod_{i \in \mathcal{C}_r \setminus \mathcal{C}_t} p_{0,i} \prod_{i' \in \mathcal{C}_t \setminus (\mathcal{C}_r \cup \mathcal{S})} p_{i',K+1} \prod_{i'' \in \mathcal{S}} (1 - p_{i'',K+1}) \Bigg[\prod_{j \in \mathcal{C}_{r,t}} p_{0,j} \prod_{j' \in \mathcal{C}_{r,t} \setminus \mathcal{S}} p_{j',K+1} + \sum_{k \in \mathcal{C}_{r,t} \setminus \mathcal{S}} \Bigg((1 - p_{0,k})(1 - p_{k,K+1}) \prod_{k' \in \mathcal{H}_{\mathcal{C}_{r,t},k}} p_{0,k'} \prod_{k'' \in \mathcal{C}_{r,t} \setminus (\mathcal{S} \cup \{k\})} p_{k'',K+1} \Bigg) \Bigg]. \tag{29}$$

After further manipulations, (29) simplifies to the following expression in the case of symmetrical networks:

$$p^{(IV)} = p^{\phi-\theta} q^{\psi-|\mathcal{S}|}(1-q)^{|\mathcal{S}|} \left[p^{\theta} + (1-p)(1-q)q^{-1} \sum_{k\in\mathcal{C}_{r,t}\setminus\mathcal{S}} p^{|\mathcal{H}_{\mathcal{C}_{r,t},k}|} \right], \quad (30)$$

where the summation that appears in (30) depends on the specific value of the state $(l_1, \ldots, l_K)$ and, hence, can not be simplified any further.

V. Buffer-Aided Selective Relaying With Load-Balancing

A. Motivation

In the case of selective-relaying with asymmetrical networks, the relays that are closer to S possess, on average, stronger S-R links and, hence, have a higher chance to be selected for reception in the first hop increasing the rate of successful arrival of packets at their buffers. On the other hand, these relays that are closer to S will be farther from D and, hence, the probability of selecting the corresponding R-D links will be low entailing a low rate of packet departure from their buffers. As a conclusion, the relays that are closer to S will suffer from packet overload where their buffers will be full most of the time. This buffer saturation will imply that the corresponding normally-strong S-R links will be unavailable and, hence, can not be selected. Consequently, potentially-weaker (but available) S-R links will be selected, thus, entailing an increase in the outage probability. This highlights the importance of implementing a load-balancing strategy as a means to even out the distribution of packets among the buffers.

B. Cooperation Strategy

The Selective-Relaying Load-Balancing (SR-LB) protocol can be implemented in the presence of inter-relay links. We assume that FSO links are established between R_k and the relays R_{k-1} and R_{k+1} (if any). For the SR-LB scheme, simultaneous transmissions take place along (i): a selected S-R link, (ii): the $K-1$ R-R links and (iii): a selected R-D link where the strongest available S-R and R-D links are selected based on (14) in a way that is completely analogous to the SR scheme. In this case, the transmit power needs to be split among $N_{\text{link}} = K+1$ links.

The inter-relay communications are managed as follows. Relay R_k alleviates its buffer occupancy by transmitting a packet to the subsequent relay R_{k+1} for $k = 1, \ldots, K-1$. Based on the assumption that $d_{0,1} \leq \cdots \leq d_{0,K}$, R_k is at a closer distance to S as compared to R_{k+1} implying that the average queue length at R_k will be greater than the average queue length at R_{k+1}. Therefore, based on the adopted assumption, activating the inter-relay links in the direction $R_1 \to R_2$, $R_2 \to R_3$, … and $R_{K-1} \to R_K$ ensures the flow of packets from the more congested buffers to the less congested buffers thus accomplishing load-balancing.

At R_{k+1}, priority will be given to the reception from S rather than R_k implying that no packet will be transmitted from R_k to R_{k+1} if $l_{k+1} \geq L-1$. In fact, for $l_{k+1} = L-1$ the single empty buffer slot will be reserved to the packet transmitted from S while for $l_{k+1} = L$ the buffer is full and no packets can be received from S and R_k. Similarly, at R_k, priority will be given to the transmission to D rather than R_{k+1} implying that no packet will be transmitted from R_k to R_{k+1} if $l_k \leq 1$. In fact, for $l_k = 1$ R_k will attempt to send the sole packet in its buffer to D rather than R_{k+1} while for $l_k = 0$ R_k can neither transmit to D nor to R_{k+1}. As a conclusion, the link R_k-R_{k+1} is considered to be available when $l_k \geq 2$ and $l_{k+1} \leq L-2$ for $k = 1, \ldots, K-1$.

C. State Transition Matrix

1) State Variations: We introduce the following flag that captures the availability of R_k-R_{k+1}:

$$\mathfrak{f}_k = \begin{cases} 1, & l_k \in \{2, \ldots, L\},\ l_{k+1} \in \{0, \ldots, L-2\}; \\ 0, & \text{otherwise.}, \end{cases} \quad (31)$$

where this link is available (resp. unavailable) if $\mathfrak{f}_k = 1$ (resp. $\mathfrak{f}_k = 0$).

The successful activation of the link R_k-R_{k+1} incurs a decrease of l_k by 1 and an increase of l_{k+1} by 1. implying that $(l_1, \ldots, l_K)$ will vary by the quantity $(-\mathbf{e}_k + \mathbf{e}_{k+1})$. Let $\mathcal{V}_{\text{LB}}$ be the set comprising all possible combinations of elements of the set $\{(-\mathbf{e}_k + \mathbf{e}_{k+1}) \mid \mathfrak{f}_k = 1\}$:

$$\begin{aligned} \mathcal{V}_{\text{LB}} = \{&(-\mathbf{e}_k + \mathbf{e}_{k+1}) \mid \mathfrak{f}_k = 1\} \cup \\ \{&(-\mathbf{e}_k + \mathbf{e}_{k+1}) + (-\mathbf{e}_{k'} + \mathbf{e}_{k'+1}) \mid \mathfrak{f}_k = \mathfrak{f}_{k'} = 1\} \cup \\ \{&(-\mathbf{e}_k + \mathbf{e}_{k+1}) + (-\mathbf{e}_{k'} + \mathbf{e}_{k'+1}) + (-\mathbf{e}_{k''} + \mathbf{e}_{k''+1}) \\ &\mid \mathfrak{f}_k = \mathfrak{f}_{k'} = \mathfrak{f}_{k''} = 1\} \cup \cdots, \end{aligned} \quad (32)$$

where $|\mathcal{V}_{\text{LB}}| = 2^{\sum_{k=1}^{K-1} \mathfrak{f}_k} - 1$. The set $\mathcal{V}_{\text{LB}}$ contains all possible additional state-variations resulting from load-balancing and that need to be added to the state-variations that result from the activation of the S-R and R-D links (i.e. SR with no load-balancing).

In a more detailed manner, consider the following state-transition: $(l_1, \ldots, l_K) \to (l_1, \ldots, l_K) + (v_1, \ldots, v_K)$. For SR (with no LB), from Section III-B.2, $(v_1, \ldots, v_K) \in \mathcal{V}_{\text{SR}}$ with:

$$\mathcal{V}_{\text{SR}} = \underbrace{\{\mathbf{0}_{1,K}\}}_{\triangleq \mathcal{V}_{\text{SR}}^{(I)}} \cup \underbrace{\{\mathbf{e}_i\ ;\ i \in \mathcal{C}_r\}}_{\triangleq \mathcal{V}_{\text{SR}}^{(II)}} \cup \underbrace{\{-\mathbf{e}_j\ ;\ j \in \mathcal{C}_t\}}_{\triangleq \mathcal{V}_{\text{SR}}^{(III)}} \cup \underbrace{\{\mathbf{e}_i - \mathbf{e}_j\ ;\ i \in \mathcal{C}_r\ ;\ j \in \mathcal{C}_t\ ;\ j \neq i\}}_{\triangleq \mathcal{V}_{\text{SR}}^{(IV)}}, \quad (33)$$

where $\mathbf{0}_{M,N}$ stands for the $M \times N$ matrix whose elements are all equal to 0.

For the SR-LB scheme, $(v_1, \ldots, v_K)$ will belong to the extended set $\mathcal{V}_{\text{SR-LB}}$ where:

$$\mathcal{V}_{\text{SR-LB}} = \left(\mathcal{V}_{\text{SR}}^{(I)} \oplus \mathcal{V}_{\text{LB}}\right) \cup \left(\mathcal{V}_{\text{SR}}^{(II)} \oplus \mathcal{V}_{\text{LB}}\right) \cup \left(\mathcal{V}_{\text{SR}}^{(III)} \oplus \mathcal{V}_{\text{LB}}\right) \cup \left(\mathcal{V}_{\text{SR}}^{(IV)} \oplus \mathcal{V}_{\text{LB}}\right), \quad (34)$$

where the set addition $\oplus$ is defined as: $\mathcal{S} \oplus \mathcal{S}' = \{s + s';\ s \in \mathcal{S},\ s' \in \mathcal{S}'\}$.

For example, consider the case of two relays. $\mathcal{V}_{\text{SR-LB}} \setminus \mathcal{V}_{\text{SR}} = \{(-1, 2), (-2, 1), (-2, 2)\}$ if $\mathfrak{f}_1 = 1$ where these three

TABLE I
SIGNALLING OVERHEADS OF THE COOPERATION PROTOCOLS

	S-R hop	R-D hop
SR	$\mathrm{R}_k \rightarrow \mathrm{S}: \begin{cases} \text{value of } G_{0,k}h_{0,k}, & \text{buffer at } \mathrm{R}_k \text{ not full;} \\ \text{nothing,} & \text{buffer at } \mathrm{R}_k \text{ full.} \end{cases}$ $\mathrm{S} \rightarrow \mathrm{R}_k$: nothing	$\mathrm{R}_k \rightarrow \mathrm{D}: \begin{cases} \text{value of } G_{k,K+1}h_{k,K+1}, & \text{buffer at } \mathrm{R}_k \text{ not empty;} \\ \text{nothing,} & \text{buffer at } \mathrm{R}_k \text{ empty.} \end{cases}$ $\mathrm{D} \rightarrow \mathrm{R}_k$: 1 bit (transmit or not)
AR	$\mathrm{R}_k \rightarrow \mathrm{S}$: $\log_2(l_k)$ bits (size of buffer) $\mathrm{S} \rightarrow \mathrm{R}_k$: 1 bit (store packet or not)	$\mathrm{R}_k \rightarrow \mathrm{D}$: nothing $\mathrm{D} \rightarrow \mathrm{R}_k$: nothing

additional state-variations follow from implementing the LB strategy.

2) Transition Probabilities: Let $f_k = 1$ if a packet is successfully transmitted along the link R$_k$-R$_{k+1}$ and $f_k = 0$ otherwise. Since $\mathfrak{f}_k$ describes the availability of this link, then $\mathfrak{f}_k = 0 \Rightarrow f_k = 0$ because no successful transmission can occur since the link is unavailable. Therefore:

$$\Pr(f_k = 0|\mathfrak{f}_k = 0) = 1; \quad \Pr(f_k = 1|\mathfrak{f}_k = 0) = 0. \tag{35}$$

On the other hand, $f_k \in \{0, 1\}$ if $\mathfrak{f}_k = 1$. In this case, $(\mathfrak{f}_k, f_k) = (1, 0)$ when the link is available for potential transmission but is in outage while $(\mathfrak{f}_k, f_k) = (1, 1)$ when the link is available and not in outage. Consequently,

$$\Pr(f_k = 0|\mathfrak{f}_k = 1) = p_{k,k+1}; \quad \Pr(f_k = 1|\mathfrak{f}_k = 1) = 1 - p_{k,k+1}, \tag{36}$$

where $p_{k,k+1}$ is the outage probability of link R$_k$-R$_{k+1}$ that can be determined from (4).

Consider the SR-LB state-transition $(l_1, \ldots, l_K) \rightarrow (l_1, \ldots, l_K) + (\upsilon_1, \ldots, \upsilon_K)$. Assuming the knowledge of the vector $(f_1, \ldots, f_K)$, the equivalent SR state-transition resulting from the activation of the S-R and R-D links alone (without the R-R links) is given by the vector:

$$(\upsilon'_1, \ldots, \upsilon'_K) = (\upsilon_1, \ldots, \upsilon_K) - \sum_{k=1}^{K-1} f_k(-\mathbf{e}_k + \mathbf{e}_{k+1}). \tag{37}$$

The two following cases follow. (i): $(\upsilon'_1, \ldots, \upsilon'_K) \in \mathcal{V}_{\text{SR}}$ given in (33). In this case, the transition $(l_1, \ldots, l_K) \rightarrow (l_1, \ldots, l_K) + (\upsilon'_1, \ldots, \upsilon'_K)$ can be tolerated by the SR scheme (with no inter-relay cooperation) and, consequently, the transition probability can be evaluated as in Section III-B.2. (ii): $(\upsilon'_1, \ldots, \upsilon'_K) \notin \mathcal{V}_{\text{SR}}$ implying that the transition $(l_1, \ldots, l_K) \rightarrow (l_1, \ldots, l_K) + (\upsilon'_1, \ldots, \upsilon'_K)$ is impossible to take place with the SR scheme implying a zero value for the transition probability.

Therefore, with the SR-LB protocol, the transition probabilities can be calculated as follows:

$$\begin{aligned} &\Pr\left((l_1, \ldots, l_K) \rightarrow (l_1, \ldots, l_K) + (\upsilon_1, \ldots, \upsilon_K)\right) \\ &\quad = \sum_{f_1=0}^{1} \cdots \sum_{f_{K-1}=0}^{1} \Pr(f_1|\mathfrak{f}_1) \cdots \Pr(f_{K-1}|\mathfrak{f}_{K-1}) \\ &\quad\quad \times p^{(\text{SR})}\left((\upsilon'_1, \ldots, \upsilon'_K)\right); \quad (\upsilon_1, \ldots, \upsilon_K) \in \mathcal{V}_{\text{SR-LB}}, \end{aligned} \tag{38}$$

where the set $\mathcal{V}_{\text{SR-LB}}$ is constructed according to (32)-(34). The flags $\{\mathfrak{f}_k\}_{k=1}^{K-1}$ are given in (31), the probabilities $\{\Pr(f_k|\mathfrak{f}_k)\}_{k=1}^{K-1}$ can be determined according to (35)-(36) while the vector $(\upsilon'_1, \ldots, \upsilon'_K)$ is defined in (37). Finally:

$$\begin{aligned} &p^{(\text{SR})}\left((\upsilon'_1, \ldots, \upsilon'_K)\right) \\ &\quad = \begin{cases} p^{(i)}, & (\upsilon'_1, \ldots, \upsilon'_K) \in \mathcal{V}_{\text{SR}}^{(i)}, \ i = I, II, III, IV; \\ 0, & \text{otherwise.}, \end{cases} \end{aligned} \tag{39}$$

where the Type-I, Type-II, Type-III and Type-IV SR transition probabilities $p^{(I)}$, $p^{(II)}$, $p^{(III)}$ and $p^{(IV)}$ are given in (18), (19), (20) and (21), respectively.

The signalling overheads of the SR and AR protocols are summarized in Table I. Concerning the SR-LB scheme, the protocol overheads along the S-R and R-D links are the same as for the SR scheme. In this case, for the R-R links, one signalling bit needs to be communicated from R$_k$ to R$_{k-1}$ indicating whether R$_k$ can receive a packet from R$_{k-1}$ or not.

VI. OUTAGE PROBABILITY AND AVERAGE PACKET DELAY

A. Stationary Distribution

The stationary distribution corresponding to the state transition matrix $\mathbf{A}$ is given by [12]:

$$\pi = \left(\mathbf{A} - \mathbf{I}_{(L+1)^K} + \mathbf{1}_{(L+1)^K,(L+1)^K}\right)^{-1} \mathbf{1}_{(L+1)^K,1}, \tag{40}$$

where $\mathbf{1}_{M,N}$ stands for the $M \times N$ matrix whose elements are all equal to 1 while $\mathbf{I}_M$ denotes the $M \times M$ identity matrix. The i-th element π_i of the $(L+1)^K$-dimensional vector π corresponds to the steady-state probability of the state $\mathfrak{N}^{-1}(i)$; in other words, the steady-state probability of having l_k packets in the k-th buffer for $k = 1, \ldots, K$ where $(l_1, \ldots, l_K) = \mathfrak{N}^{-1}(i)$. Vector π constitutes the key component in evaluating the outage and delay performance of the network.

B. System Outage Probability

An outage event occurs when there is no change in the buffer status due to the failure of the S-R and R-D hops. In this case, the network is inaccessible due to the unavailability of both hops. Consequently, the network outage probability can be expressed as:

$$\begin{aligned} P_{\text{out}} = \sum_{i=1}^{(L+1)^K} \pi_i P_{\mathcal{C}_r} Q_{\mathcal{C}_t} = \sum_{i=1}^{(L+1)^K} \pi_i \prod_{\substack{k=1 \\ l_k \neq L}}^{K} p_{0,k} \prod_{\substack{k'=1 \\ l_{k'} \neq 0}}^{K} p_{k',K+1}; \\ (l_1, \ldots, l_K) = \mathfrak{N}^{-1}(i), \end{aligned} \tag{41}$$

where the sets $\mathcal{C}_r$ and $\mathcal{C}_t$ are determined from the state $(l_1, \ldots, l_K) = \mathfrak{N}^{-1}(i)$ according to (6)-(7). The probabilities $P_{\mathcal{C}_r}$ and $Q_{\mathcal{C}_t}$ are defined in (9) and (10), respectively, and assume the same expressions with SR and AR (only the value of N_{link} in (4) will change).

C. Average Packet Delay

Because of buffering at the relays, the packets transmitted by the source will reach the destination with a certain delay. The average packet delay can be split into two parts $\mathrm{E}[D] = \mathrm{E}[D_s] + \mathrm{E}[D_r]$ where E[.] stands for the time-averaging operator while $\mathrm{E}[D_s]$ and $\mathrm{E}[D_r]$ stand for the average delays at the source and relays, respectively.

According to Little's law [19], the average delay at the relays can be calculated from $\mathrm{E}[D_r] = \frac{\bar{L}}{\eta_s}$ where $\bar{L}$ stands for the average queue length that can be calculated as follows:

$$\bar{L} = \sum_{i=1}^{(L+1)^K} \pi_i \sum_{k=1}^{K} l_k; \quad (l_1, \ldots, l_K) = \mathfrak{N}^{-1}(i). \tag{42}$$

The parameter η_s stands for the input throughput at the relays which is the same as the output throughput from the source. This throughput depends on the conditions of the S-R channels and on the availability of the relays' buffers. It can be calculated as follows:

$$\eta_s = \sum_{i=1}^{(L+1)^K} \pi_i \left(1 - P_{\mathcal{C}_r}\right), \tag{43}$$

where $\mathcal{C}_r$ is determined from the state $(l_1, \ldots, l_K) = \mathfrak{N}^{-1}(i)$ according to (6) while the term $1 - P_{\mathcal{C}_r}$ stands for the probability that at least one of the available S-R links is not in outage.

Since the source is assumed to have infinite data, $\mathrm{E}[D_s]$ depends on how frequently the first S-R hop is activated and can be calculated from $\mathrm{E}[D_s] = \frac{1}{\eta_s} - 1$ where $1/\eta_s$ stands for the average number of attempts needed to successfully transmit one packet along the first hop and, thus, decrease the source delay by 1.

Combining the above equations results in the following expression of the average packet delay:

$$\mathrm{E}[D] = \frac{1 + \sum_{i=1}^{(L+1)^K} \pi_i \sum_{k=1}^{K} l_k}{\sum_{i=1}^{(L+1)^K} \pi_i \left(1 - \prod_{\substack{k=1 \\ l_k \neq L}}^{K} p_{0,k}\right)} - 1;$$
$$(l_1, \ldots, l_K) = \mathfrak{N}^{-1}(i). \tag{44}$$

On the other hand, the infinite buffer employed by the source node is stable only if η_s exceeds the rate at which the packets arrive at the source's buffer. As far as the relays' queues stabilities are concerned, it is important to note that the relays' buffers are both rate and mean rate stable as per the following explanation. According to [20], if $l(t)$ designates the instantaneous length of a queueing system defined over time slots $t \in \{0, 1, 2, ...\}$, then the queue is considered to be rate stable when $\lim_{t\to\infty} \frac{l(t)}{t} = 0$ with probability 1. Moreover, the queue is said to be mean rate stable if $\lim_{t\to\infty} \frac{\bar{L}}{t} = 0$, where $\bar{L} = \mathrm{E}[l(t)]$ is the expected relay buffer length. In the context of the proposed buffer-aided relay-assisted FSO system, relays' buffers have a finite capacity of L. This implies that $l(t) \leq L$ $\forall t$ and, consequently, $\bar{L} \leq L$. Therefore, both $\lim_{t\to\infty} \frac{l(t)}{t}$ as well as $\lim_{t\to\infty} \frac{\bar{L}}{t}$ evaluate to 0 (since the numerators are finite) confirming the rate and mean rate stability of the relays' buffers.

D. Advantages of the Proposed Protocols

The cooperation strategies proposed in this paper were developed with a view to maximizing the throughput of the relay-assisted network which is a direct consequence of the minimization of the system outage probability.

For the selective schemes where only a single link is activated in any given time slot, the optimization study conducted in [21] provides fundamental guidelines about throughput maximization. Even though this study was solely concerned with the case of HD relays, its findings can be extended to our investigated system where up to two different relays can be involved in packet transmission/reception in each time slot. Reference [21, Th. 1] highlights that the maximum achievable S-to-D rate can be realized if the S-R link or R-D link with the maximum associated rate is activated at any time. Note that our proposed BA SR strategy matches the optimization requirements stipulated in the aforementioned theorem through the simultaneous activation of the strongest available S-R and R-D links.

Regarding the proposed AR protocol, [22] and many of the references therein studied a parallel queueing model that accurately captures the main characteristics of the investigated relay-assisted system. These studies proved that the Shortest Queue Policy (SQP) is the best transmission decision that can be made by the source node. More specifically, it has been shown that the throughput can be maximized when the source transmits its packet to the queue having the smallest number of packets. This transmission policy is clearly embodied in the proposed AR strategy where out of all the copies of the same packet received by the relays in any given time slot, only the relay having the shortest queue is required to retain a copy of the packet.

VII. Numerical Results

The refractive index structure constant and the attenuation constant are set to $C_n^2 = 1.7 \times 10^{-14}$ m$^{-2/3}$ and $\sigma = 0.44$ dB/km. We also fix $\eta = 1$ and $\lambda = 1550$ nm. Results show the variations of the outage probability and average delay as a function of the average received electrical SNR along the direct S-D link (in the case of non-cooperative transmissions). This average SNR takes the value $\overline{\gamma}_{0,K+1} = \frac{\eta^2}{N_0}$ that can be obtained by setting $G_{0,K+1} = 1$ and $N_{\text{link}} = 1$ in (2) while observing that $\mathrm{E}[h_{0,K+1}] = 1$. In all scenarios, the distance between S and D is assumed to be $d_{0,K+1} = 3$ km. The FSO network will be parameterized by two distances d_1 and d_2 as follows. For symmetrical networks, all relays are assumed to be at a distance d_1 from the source and d_2 from the destination. For asymmetrical networks, we set $(d_{0,1}, d_{1,K+1}) = (d_1, d_2)$ and $(d_{0,K}, d_{K,K+1}) = (d_2, d_1)$ while the remaining relays $\mathrm{R}_2, \ldots, \mathrm{R}_{K-1}$ are placed equidistantly between R_1 and R_K. An in-house custom-built Java-based discrete event simulator was developed for the purpose of verifying the validity of the mathematical models delineated earlier.

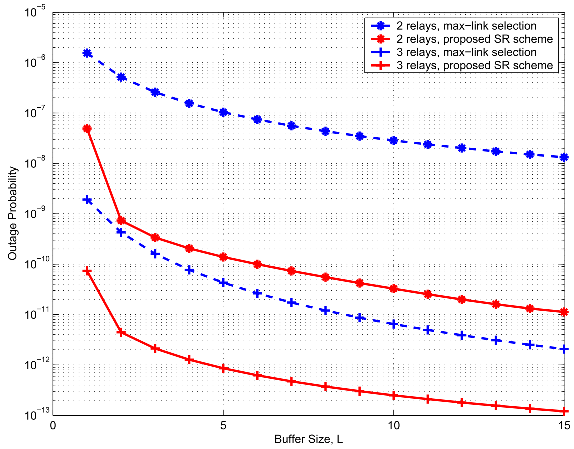

Fig. 2. Outage probability of the proposed SR scheme versus the *max-link* selection [12] at a SNR of 15 dB. A symmetrical network is considered with $d_1 = 2$ km and $d_2 = 1.5$ km.

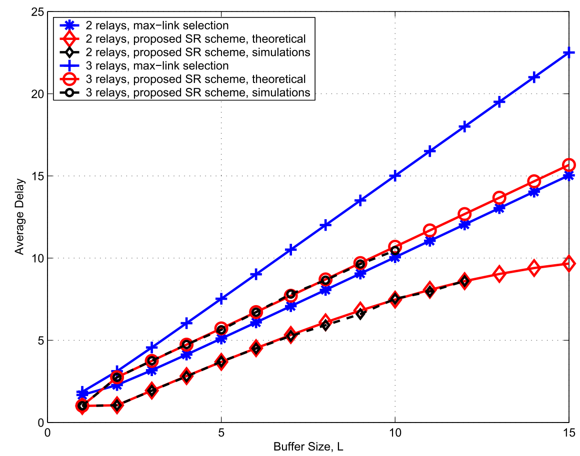

Fig. 3. Average packet delay of the proposed SR scheme versus the *max-link* selection [12] at a SNR of 15 dB. A symmetrical network is considered with $d_1 = 2$ km and $d_2 = 1.5$ km.

First, we compare the proposed SR scheme with the *max-link* selection protocol [12] thus highlighting the impact of simultaneously activating the best S-R and R-D links. A symmetrical network is considered with $(d_1, d_2) = (2, 1.5)$ km at a SNR of 15 dB in the cases of 2 and 3 relays. The outage performance is highlighted in Fig. 2 while the average delay is shown in Fig. 3 as a function of the buffer size L. While the reported outage probabilities are very small to be reconstructed numerically, the delays obtained by simulations are consistent with the theoretical delays. Results highlight the superiority of the SR scheme where enhanced performance levels and reduced delays are observed for all buffer sizes. These results are expected since the SR scheme is better tailored to the nature of FSO transmissions that result in no interference. The simultaneous activation of two links in each time slot (versus one link for *max-link* selection) has a predominant effect on the delay where at $L = 15$, for example, the average delay is reduced by around 5 (resp. 7) time slots for $K = 2$ (resp. $K = 3$). Figures 2 and 3 also highlight the tradeoff that exists between the outage probability and average delay. In fact, the outage performance is enhanced by increasing the values of K and L at the expense of increased delays. In fact, as K increases a given S-R or R-D link has a lower chance to be selected thus increasing the delay while buffers with bigger sizes result in longer waiting times to exit the queue. Finally, it is worth noting that comparing the average delays with [12] is not completely fair since [12] was designed for half-duplex (HD) broadcast RF communications not for full-duplex (FD) directive FSO communications. However, this comparison is carried out given the absence of any existing buffer-aided FSO relaying scheme that we can use for benchmarking. This comparison does not show the limitation of [12] that is primarily designed under different construction constraints; on the contrary, this comparison is provided for the sake of highlighting the gains that can be entailed from the absence of interference and presence of FD relays. In other words, results in Fig. 3 emphasize on the advantages that can be harvested from properly exploiting the additional degrees of freedom that are offered by the nature of FSO communications.

In Fig. 4, we compare the proposed SR and AR schemes in the context of a symmetrical network with $(d_1, d_2) = (1.5, 1.8)$ km. We also show the performance of the non-buffer-aided parallel-relaying (PR) and *max-min* schemes proposed in [1] and [6], respectively. Since these schemes do not use buffers, they both result in a zero delay. While the *max-min* scheme activates the strongest end-to-end path and is capable of achieving higher performance levels at the expense of an increased complexity since the full CSI needs to be acquired, the PR scheme is appealing because of its simplicity since all S-R and R-D links are activated without the need of the CSI at the expense of reduced performance levels. It is worth noting that the performance of the benchmark schemes [1] and [6] can be enhanced by implementing temporal diversity methods and/or joint encoding/decoding schemes. However, some extensions might require adding buffers and will eventually incur some delays. A comparison between the proposed BA schemes and the extension of the benchmark schemes needs to be carried out at the same tolerated average delay levels and falls beyond the scope of this paper. As expected, the SR scheme that operates under perfect acquisition of the CSI achieves the best performance levels as shown in Fig. 4.a and Fig. 4.b for $K = 2$ and $K = 3$, respectively. However, the corresponding delay curves presented in Fig. 4.c and Fig. 4.d show that these performance gains are associated with significant delays that increase very rapidly with the buffer size L. This emphasizes the interest of the AR scheme as a delay-efficient alternative to the SR scheme where the corresponding average packet delays are significantly smaller than those attained by the SR protocol for practical SNR values exceeding 5 dB. Results in Fig. 4 also highlight the impact of the number of relays on the system performance. For example, increasing the number of relays from two to three with the SR scheme for $L = 5$ results in a performance gain in the order of 2.5 dB at an outage probability of 10^{-5}.

Results in Fig. 4 highlight an important particularity of the AR scheme that resides in the fact that increasing the

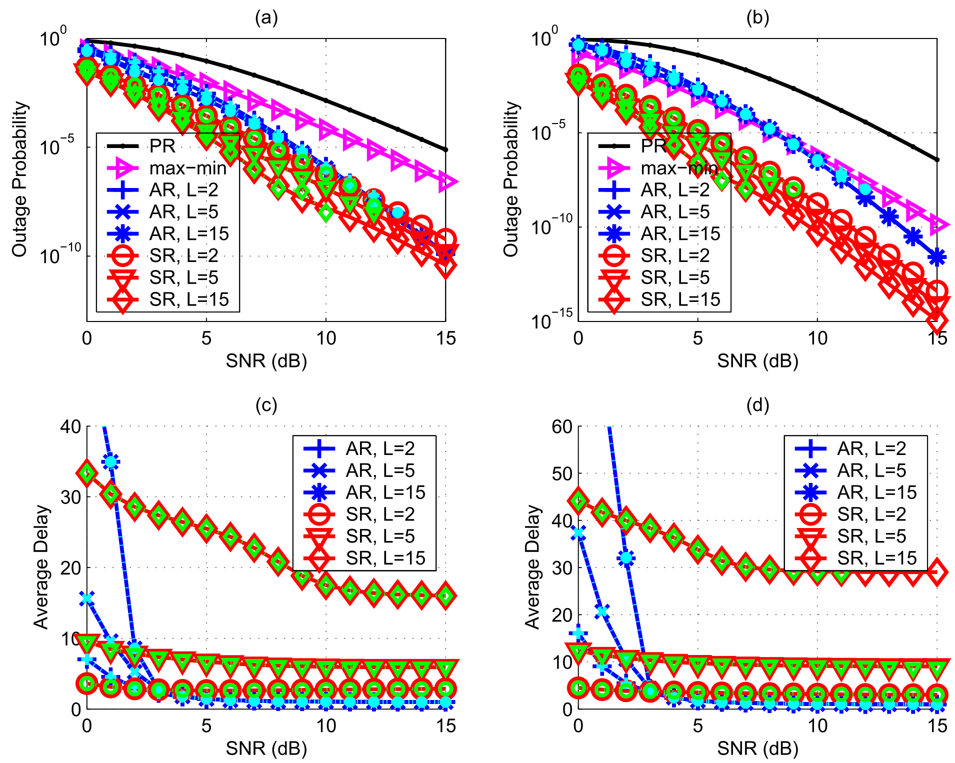

Fig. 4. Performance of the SR scheme, AR scheme, parallel-relaying (PR) scheme [1] and the *max-min* scheme [6]. A symmetrical network is considered with $d_1 = 1.5$ km and $d_2 = 1.8$ km. (a)-(b): Outage probability for $K = 2$ and $K = 3$, respectively. (c)-(d): Average packet delay for $K = 2$ and $K = 3$, respectively. Dotted lines correspond to the numerical results.

buffer size beyond 2 has no meaningful impact on the system performance for SNR values exceeding 5 dB. This renders the AR protocol an appealing solution for FSO relaying systems with limited buffer sizes where the optimal asymptotic performance ($L \to \infty$) can be achieved by a practical buffer size not exceeding two. This is related to the fact that at most one relay is filling at a time while all relays are attempting to empty their buffers all of the time. For example, an analysis of the buffers' occupancy at a SNR of 10 dB demonstrates that $\Pr(l_k > 2) = 0$ for $k = 1, \ldots, K$ and, thus, increasing the buffer size beyond two does not affect the system performance. This validates the fact that the outage probability curves and delay curves of the AR scheme in Fig. 4 with $L = 2$, $L = 5$ and $L = 15$ almost overlap for SNR values exceeding 5 dB. For example, for $L = 5$, the buffer's occupancy analysis shows that the third, fourth and fifth slots are never filled and, hence, these slots can be removed without affecting the system performance. Fig. 4 also highlights that, even with small buffer sizes, the AR protocol is capable of achieving significant performance gains with respect to the non-buffer-aided DF parallel-relaying (PR) scheme [1]. Once again, Fig. 4 demonstrates the close match between the theoretical and numerical results.

Results in Fig. 4.b also show that the AR and *max-min* schemes exhibit comparable outage performances for SNRs below 10 dB. For this SNR range, the advantage of the proposed AR scheme over the *max-min* scheme resides in the possibility of implementing the former scheme in the absence of CSI. Acquiring the full CSI incurs additional levels of complexity to the system where training symbols need to be transmitted along all links for the sake of estimating the channels and, at a second time, the estimated channels need to be fed back to the source node. Moreover, in practical systems, any error in the channel estimation will incur performance losses with respect to the curves reported in Fig. 4.b where perfect CSI acquisition is assumed. In this context, equipping each relay with a small buffer whose storage capability does not exceed two packets circumvents the challenges and limitations of ideal channel estimation. For SNRs exceeding 10 dB, Fig. 4.b shows that the AR scheme results in enhanced diversity orders where the gap between the AR and *max-min* schemes increases with the SNR.

Fig. 5 highlights the impact of load-balancing where the SR and SR-LB schemes are compared with $L = 5$ for an asymmetrical network configuration with $(d_1, d_2) = (1.25, 2.5)$ km. In other words, the set of distances $\{(d_{0,k}, d_{k,K+1})\}_{k=1}^{K}$ takes the values $\{(1.25, 2.5), (2.5, 1.25)\}$, $\{(1.25, 2.5), (1.5, 1.5), (2.5, 1.25)\}$ and $\{(1.25, 2.5), (1.3, 1.8), (1.8, 1.3), (2.5, 1.25)\}$ for $K = 2$, $K = 3$ and $K = 4$, respectively. Results highlight the importance of load balancing where significant improvements in the outage performance are observed for different numbers of relays. The marginal losses at low SNRs result from allocating a fraction of the transmit power to the R-R links ($N_{\text{link}} = 2$ for SR versus $N_{\text{link}} = K + 1$ for SR-LB). Let $\mathbf{\Pi}^{(k)} = [\Pi_0^{(k)}, \ldots, \Pi_L^{(k)}]$ denote the steady-state

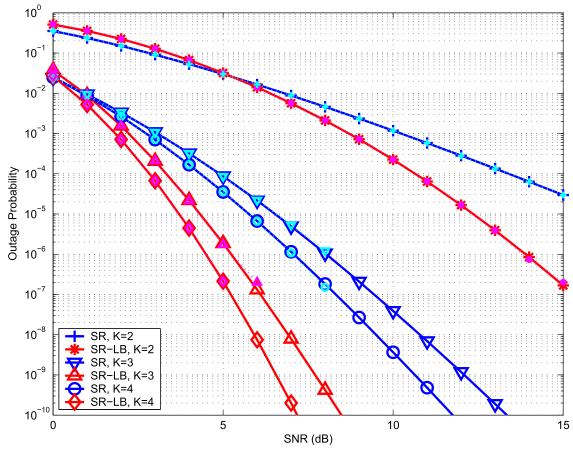

Fig. 5. The SR scheme versus the SR-LB scheme for an asymmetrical network with $d_1 = 1.25$ km and $d_2 = 2.5$ km. Dotted lines correspond to the numerical results.

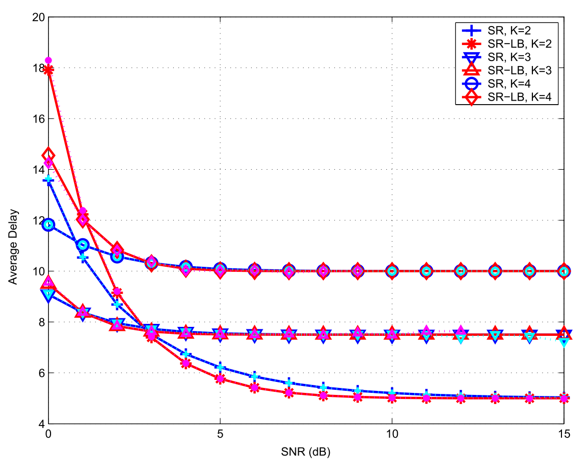

Fig. 6. Average packet delays for the simulation setup in Fig. 5. Dotted lines correspond to the numerical results.

probability distribution of the number of packets in the k-th relay's buffer. For the SR scheme with $K = 2$ at a SNR of 10 dB, $\mathbf{\Pi}^{(1)} = [0, 0, 0, 0.02, 0.5, 0.48]$ and $\mathbf{\Pi}^{(2)} = [0.48, 0.5, 0.02, 0, 0, 0]$ indicating that the buffer at R_1 is full most of the time while the buffer at R_2 is empty most of the time negatively impacting the accessibility of the network. When the proposed load-balancing scheme is applied, the above distributions take the following values $\mathbf{\Pi}^{(1)} = [0, 0.04, 0.3, 0.4, 0.23, 0.03]$ and $\mathbf{\Pi}^{(2)} = [0.03, 0.23, 0.4, 0.3, 0.04, 0]$. In other words, the distribution $\mathbf{\Pi}^{(1)}$ is shifted towards smaller buffer sizes (the average queue length drops from 4.46 to 2.9) while the distribution $\mathbf{\Pi}^{(2)}$ is shifted towards larger buffer sizes (the average queue length increases from 0.53 to 2) thus improving the availability to the S-R and R-D links. The corresponding average packet delays are shown in Fig. 6 where the results show that the delays introduced by the SR and SR-LB schemes are roughly the same for average-to-large SNR values. For low SNRs, the SR-LB scheme results in increased delays since, given the high unavailability of the network links, the packet might move among many buffers before being delivered to D. In this context, the negative impact of the increased value of N_{link} manifests mainly at low SNRs. Evidently, the delays increase with the number of relays whether for the SR or SR-LB schemes.

VIII. Conclusion

Equipping the FSO relays with buffers constitutes an additional degree of freedom that significantly enhances the performance at the expense of increased delays. Handling the best-relay selection in an FD manner conjointly reduces the outage probability and delay by several orders of magnitude. On the other hand, concurrently activating all available FSO links alleviates the signaling complexity, achieves the best reported delays and constitutes an appealing solution for small buffer sizes that practically yield the same performance as infinite-size buffers. Finally, for asymmetrical networks, exploiting the potential presence of the relay-relay links for balancing the loads of the different buffers results in phenomenal enhancements in the performance. This work constitutes an essential step in the direction of motivating the introduction of buffers to relay-assisted FSO systems. Future studies can build on this work to explore many other interesting aspects of the proposed buffer-aided relay-assisted architecture, including, among others, its associated diversity order and the effect of possible relay mobility on the system performance.

Appendix

A. Type-I Transition Probability

The buffer sizes of the relays with empty buffers do not change only if the corresponding S-R links are in outage with probability $\prod_{k\in\mathcal{C}_r\setminus\mathcal{C}_t} p_{0,k}$. Similarly, the buffer sizes of the relays with full buffers do not change only if the corresponding R-D links are in outage with probability $\prod_{k'\in\mathcal{C}_t\setminus\mathcal{C}_r} p_{k',K+1}$. The above probabilities simplify to $p^{\phi-\theta}$ and $q^{\psi-\theta}$ for symmetrical networks.

Consider now the relays that can receive and transmit (i.e. elements of the set $\mathcal{C}_{r,t}$ in (8)). The buffer sizes of these relays do not change only in one of the two following mutually-exclusive scenarios. (i): The corresponding S-R links are all in outage. In this case, nothing is received by these relays and, hence, the R-D links should be in outage as well so that nothing will be transmitted and the sizes of the buffers remain the same. The corresponding probability is $\prod_{i\in\mathcal{C}_{r,t}} p_{0,i}p_{i,K+1}$ and simplifies to $p^{\theta}q^{\theta}$ for symmetrical networks. (ii): At least one of the S-R links is not in outage. In this case, the packet will be kept at one relay (the one with the highest priority) while it is dropped from the remaining relays. To keep the same buffer size, this relay that did not drop the packet must forward it to D in the next hop while the remaining R-D hops must be in outage. Therefore, the corresponding probability can be written as:

$$\sum_{i\in\mathcal{C}_{r,t}}(1-p_{0,i})\prod_{i'\in\mathcal{H}_{\mathcal{C}_{r,t},i}} p_{0,i'}\ (1-p_{i,K+1})\prod_{j\in\mathcal{C}_{r,t}\setminus\{i\}} p_{j,K+1}, \quad (45)$$

where (24) was invoked. In the case of symmetrical channels, this probability simplifies to $(1-p^{\theta})(1-q)q^{\theta-1}$ where $(1-p^{\theta})$

is the probability that at least one of the S-R links is not in outage. Combining the above probabilities results in the expression given in (25).

B. Type-II Transition Probability

Since the buffer size of R_i increased by 1, then R_i has successfully decoded the packet (i.e. link S-R_i is not in outage) while all remaining relays with higher priority suffer from outage (otherwise, the packet will be eventually dropped from R_i). In this scenario, R_i (resp. R_j for $j \neq i$) must not be able to retransmit the packet; otherwise, the buffer size will drop to 0 (resp. -1) implying that all available R-D links must be in outage. It is worth noting that the relays with a priority lower than R_i are not considered in (27) since even if these relays successfully receive the packet, it will be eventually dropped since R_i (with a higher priority) has successfully received the packet as well.

C. Type-IV Transition Probability

To derive the Type-IV transition probability, the set $\mathcal{C}_r \cup \mathcal{C}_t$ is partitioned as follows:

$$\mathcal{C}_r \cup \mathcal{C}_t = \underbrace{[\mathcal{C}_r \backslash \mathcal{C}_t]}_{\triangleq \mathcal{C}_1} \cup \underbrace{[\mathcal{C}_t \backslash (\mathcal{C}_r \cup \mathcal{S})]}_{\triangleq \mathcal{C}_2} \cup \underbrace{[\mathcal{S} \backslash \mathcal{C}_{r,t}]}_{\triangleq \mathcal{C}_3} \cup \underbrace{\mathcal{C}_{r,t}}_{\triangleq \mathcal{C}_4}. \quad (46)$$

Based on the partitioning in (46), the Type-IV transition probability can be written as:

$$p^{(IV)} = p_1^{(IV)} \times p_2^{(IV)} \times p_3^{(IV)} \times p_4^{(IV)}, \quad (47)$$

where $p_n^{(IV)}$ is associated with the set $\mathcal{C}_n$ in (46).

Elements of $\mathcal{C}_1$ can receive but not transmit and have their buffer sizes unchanged (since $\mathcal{S} \subset \mathcal{C}_t$). Consequently, the corresponding S-R links must be in outage and $p_1^{(IV)} = \prod_{i \in \mathcal{C}_r \backslash \mathcal{C}_t} p_{0,i}$.

Elements of $\mathcal{C}_2$ can transmit but not receive and have their buffer sizes unchanged. Consequently, the corresponding R-D links must be in outage and $p_2^{(IV)} = \prod_{i' \in \mathcal{C}_t \backslash (\mathcal{C}_r \cup \mathcal{S})} p_{i',K+1}$.

Elements of $\mathcal{C}_3$ can transmit but not receive and have their buffer sizes decrease by 1. Consequently, the corresponding R-D links must not be in outage and $p_3^{(IV)} = \prod_{i'' \in \mathcal{S} \backslash \mathcal{C}_{r,t}} (1 - p_{i'',K+1})$.

Now, $\mathcal{C}_4$ that can be partitioned as $\mathcal{C}_{r,t} = [\mathcal{C}_{r,t} \backslash \mathcal{S}] \cup [\mathcal{C}_{r,t} \cap \mathcal{S}]$. One of the two following mutually-exclusive scenarios might arise and $p_4^{(IV)}$ can be written as $p_{4,1}^{(IV)} + p_{4,2}^{(IV)}$. Scenario 1: All the corresponding S-R links are in outage with probability $\prod_{j \in \mathcal{C}_{r,t}} p_{0,j}$. Now, since nothing has been received at relays in $\mathcal{C}_{r,t} \backslash \mathcal{S}$ while their buffer sizes remained the same, then the corresponding R-D links must be in outage with probability $\prod_{j' \in \mathcal{C}_{r,t} \backslash \mathcal{S}} p_{j',K+1}$. Similarly, since nothing has been received at relays in $\mathcal{C}_{r,t} \cap \mathcal{S}$ while their buffer sizes decreased by 1, then the corresponding R-D links must not be in outage with probability $\prod_{j'' \in \mathcal{C}_{r,t} \cap \mathcal{S}} (1 - p_{j'',K+1})$. Consequently:

$$p_{4,1}^{(IV)} = \prod_{j \in \mathcal{C}_{r,t}} p_{0,j} \prod_{j' \in \mathcal{C}_{r,t} \backslash \mathcal{S}} p_{j',K+1} \prod_{j'' \in \mathcal{C}_{r,t} \cap \mathcal{S}} (1 - p_{j'',K+1}). \quad (48)$$

Scenario 2: At least one of the relays in $\mathcal{C}_{r,t}$ has successfully decoded the packet. Despite the successful detection by potentially more than one relay, the AR protocol ensures that only one relay will keep the packet. Now, this relay should be in $\mathcal{C}_{r,t} \backslash \mathcal{S}$ since the buffer sizes of relays in $\mathcal{C}_{r,t} \cap \mathcal{S}$ decreased by one implying that no packet is kept from the source side. Consequently, $p_{4,2}^{(IV)}$ can be written as:

$$p_{4,2}^{(IV)} = \sum_{k \in \mathcal{C}_{r,t} \backslash \mathcal{S}} \Bigg[(1 - p_{0,k}) \prod_{k' \in \mathcal{H}_{\mathcal{C}_{r,t},k}} p_{0,k'} \ (1 - p_{k,K+1}) \times \prod_{k'' \in \mathcal{C}_{r,t} \backslash (\mathcal{S} \cup \{k\})} p_{k'',K+1} \prod_{k''' \in \mathcal{C}_{r,t} \cap \mathcal{S}} (1 - p_{k''',K+1}) \Bigg], \quad (49)$$

where the first two terms follow from (24) reflecting the fact that R_k did not drop the source packet ($k \in \mathcal{C}_{r,t} \backslash \mathcal{S}$). In this case, R_k should forward a packet to D since its buffer size did not change ($k \notin \mathcal{S}$) implying that the link R_k-D is not in outage yielding the third term in (49). Now, for the remaining relays in $\mathcal{C}_{r,t} \backslash \mathcal{S}$, the corresponding R-D links must be in outage (since the buffer sizes did not change) which results in the fourth term in (49). The fifth term follows since relays in $\mathcal{C}_{r,t} \cap \mathcal{S}$ have their buffer sizes decrease by 1 and, hence, the corresponding R-D links must not be in outage.

Replacing the different probabilities in (47), and after some manipulations, the Type-IV transition probability takes the expression given in (29).

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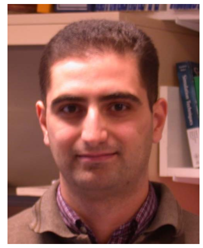
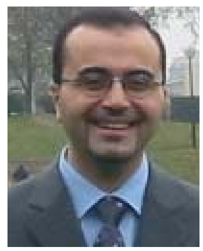


Chadi Abou-Rjeily (M'07–SM'13) received the B.E. degree in electrical engineering from the Lebanese University, Roumieh, Lebanon, in 2002, and the M.S. and Ph.D. degrees in electrical engineering from the École Nationale Supérieure des Télécommunications, Paris, France, in 2003 and 2006, respectively. From 2003 to 2007, he was a Research Fellow with the Laboratory of Electronics and Information Technology of the French Atomic Energy Commission. He is currently an Associate Professor with the Department of Electrical and Computer Engineering, Lebanese American University, Byblos, Lebanon. His research interests are in code construction and transceiver design for wireless communication systems.



Wissam Fawaz (M'07–SM'13) received the B.E. degree (Hons.) in computer engineering from the Lebanese University in 2001, the M.S. degree (Hons.) in network and information technology from the University of Paris VI in 2002, and the Ph.D. degree (Hons.) in network and information technology with from the University of Paris XIII in 2005. He is currently an Associate Professor of computer engineering with the Lebanese American University. His research interests are in the areas of delay/disruption tolerant networks and queuing theory. He was a recipient of the French Ministry of Research and Education Scholarship for distinguished students in 2002 and the Fulbright Award in 2008. He is currently serving as an Associate Editor of the IEEE COMMUNICATIONS LETTERS.