Impact of the Direct Link on the Performance of Single-Relay Buffer-Aided FSO Communications

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***Abstract*—This work targets the performance analysis of buffer-aided (BA) Free-Space Optical (FSO) relaying with a single decode-and-forward (DF) relay. In particular, we study the impact of activating the direct source-destination link on the system performance. Based on a Markov chain analysis, we derive closed-form expressions for the system outage probability (OP) and average packet delay (APD). Moreover, we carry out an asymptotic analysis and evaluate the achievable diversity order. Results show that while activating the direct link reduces the APD in all scenarios, this activation might either increase or decrease the diversity order depending on the relay position.**

***Index Terms*—Free space optics, FSO, relaying, buffer, asymptotic analysis, relay placement, diversity gain.**

I. Introduction

Free-Space Optical (FSO) communication represents a potential remedy to the Radio Frequency (RF) spectrum crunch problem. FSO systems enjoy a wide range of possible applications ranging from their ability to interconnect remote buildings in corporate and university campuses to disaster recovery [1]. Nonetheless, FSO systems are known to be deeply sensitive to weather conditions. This has triggered a thorough exploration of the benefits of cooperative diversity in overcoming the atmospheric impairments on the FSO signals. Cooperative communication revolves around the deployment of relay nodes between the source and the destination nodes with a view of mitigating the distance-dependent atmospheric-induced fading [2]. In this particular regard, most of the existing studies on cooperative communication were built upon the assumption of buffer-free cooperative relaying [3]–[5]. However, recent studies highlighted the benefits resulting from Buffer-Aided (BA) relaying, thus sparking interest in the investigation of BA cooperative relaying in the context of both RF systems [6]–[9] and FSO systems [10]–[14]. BA relaying is developed around the assumption that relays are equipped with data queues, which can be exploited to hold packets when the relay to destination link is not favorable for data transfer. This presents the advantage of enhancing the throughput of the BA system as compared to the buffer-free one.

The authors of [6]–[9] investigated BA cooperation in relay-assisted RF systems. More specifically, [6] introduced the so-called max-link protocol, whereby the time axis is slotted such that each time slot is dedicated for either source (S) to relay (R) transmission or R to destination (D) transmission. In this context, the availability of the system is improved by transmitting information over the link having the best condition among all available S-R and R-D links. The max-link protocol, which was initially designed for Decode-and-Froward (DF) cooperation in [6], was extended later to Amplify-and-Forward (AF) cooperation in [7]. Additional improvements to this protocol were introduced afterwards in [8], [9].

Consideration was given only recently to BA cooperation in the context of FSO as well as hybrid FSO/RF systems. In particular, a link allocation strategy was provided in [10] and then improved in [11] for multiuser hybrid RF and mixed FSO/RF BA networks. The authors of [12] investigated BA relay selection in the context of hybrid FSO/RF links when the relays employ infinite size queues. Lastly, the authors of [13] and [14] considered BA cooperative relaying in the presence of multiple relays having finite size buffers in the context of parallel and serial relay-assisted FSO systems, respectively.

Unlike the previously surveyed studies on BA cooperative relaying, this paper evaluates both analytically as well as via simulation the impact that the presence of a direct S-D link may have on the performance of BA relay-assisted FSO communication system. This is especially true since the existing relevant studies overlooked the possibility of having a direct link connecting the source node to the destination node. Particularly, important insights into the performance of a system encompassing a direct S-D link in terms of outage probability (OP), average packet delay (APD), and achievable diversity order are provided. Both OP and APD are studied precisely through a Markov chain framework while the diversity order is evaluated asymptotically.

II. System Model and Cooperation Protocol

Consider a BA FSO network consisting of a source (S), destination (D) and one relay (R). The relay is equipped with a buffer of finite size L packets and is placed at a distance d_{SR} from S and d_{RD} from D. The length of the direct link S-D is denoted by d_{SD}. We consider intensity-modulation with direct-detection (IM/DD) FSO communications in the case of background-noise limited receivers corrupted by additive Gaussian noise. Under the above assumptions, the outage probability along the S-D link can be determined from [13]:

$$p_{\text{SD}} = \frac{1}{\Gamma(\alpha_{\text{SD}})\Gamma(\beta_{\text{SD}})} G_{1,3}^{2,1}\left[\frac{\alpha_{\text{SD}}\beta_{\text{SD}}}{P_M/N_l}\middle|\begin{matrix}1\\ \alpha_{\text{SD}},\beta_{\text{SD}},0\end{matrix}\right], \quad (1)$$

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where we assume that the turbulence-induced atmospheric scintillation is described by the gamma-gamma model. In (1), $\Gamma(\cdot)$ stands for the gamma function while $G_{p,q}^{m,n}[.]$ is the Meijer G-function [13]. $\alpha_{\rm SD}$ and $\beta_{\rm SD}$ stand for the parameters of the gamma-gamma distribution associated with the direct link. The parameters can be written as: $\alpha_{\rm SD} \triangleq \alpha(d_{\rm SD})$ and $\beta_{\rm SD} \triangleq \beta(d_{\rm SD})$ where $\alpha(d) = \left[\exp\left(0.49\sigma_R^2(d)/(1+1.11\sigma_R^{12/5}(d))^{7/6}\right)-1\right]^{-1}$ and $\beta(d) = \left[\exp\left(0.51\sigma_R^2(d)/(1+0.69\sigma_R^{12/5}(d))^{5/6}\right)-1\right]^{-1}$ where the Rytov variance depends on the link distance d through the relation $\sigma_R^2(d) = 1.23C_n^2 k^{7/6} d^{11/6}$ with k and C_n^2 denoting the wave number and refractive index structure parameter, respectively.

In (1), P_M stands for the optical power margin that is normalised by the total number of links N_l. The normalisation follows from evenly splitting the transmit power among all available FSO links (in the absence of channel state information) so that the cooperative scheme transmits the same power as the point-to-point non-cooperative scenario. When S-D is activated, $N_l = 3$ following from splitting the power among the S-D, S-R and R-D links. On the other hand, when S-D is not activated, $N_l = 2$ since only the S-R and R-D links will be activated in this case.

Similar to (1), the outage probabilities along the indirect S-R and R-D links are given by:

$$p_{\rm SR} = \frac{1}{\Gamma(\alpha_{\rm SR})\Gamma(\beta_{\rm SR})} G_{1,3}^{2,1}\left[\frac{\alpha_{\rm SR}\beta_{\rm SR}}{G_{\rm SR}P_M/N_l}\middle|\begin{matrix}1\\ \alpha_{\rm SR},\beta_{\rm SR},0\end{matrix}\right] \quad (2)$$

$$p_{\rm RD} = \frac{1}{\Gamma(\alpha_{\rm RD})\Gamma(\beta_{\rm RD})} G_{1,3}^{2,1}\left[\frac{\alpha_{\rm RD}\beta_{\rm RD}}{G_{\rm RD}P_M/N_l}\middle|\begin{matrix}1\\ \alpha_{\rm RD},\beta_{\rm RD},0\end{matrix}\right], \quad (3)$$

where $(\alpha_{\rm SR},\beta_{\rm SR}) \triangleq (\alpha(d_{\rm SR}),\beta(d_{\rm SR}))$ and $(\alpha_{\rm RD},\beta_{\rm RD}) \triangleq (\alpha(d_{\rm RD}),\beta(d_{\rm RD}))$. The gains $G_{\rm SR}$ and $G_{\rm RD}$ follow since the S-R and R-D links might be shorter than the direct link. $G_{\rm SR} = \left(\frac{d_{\rm SD}}{d_{\rm SR}}\right)^2 e^{-\sigma(d_{\rm SR}-d_{\rm SD})}$ and $G_{\rm RD} = \left(\frac{d_{\rm SD}}{d_{\rm RD}}\right)^2 e^{-\sigma(d_{\rm RD}-d_{\rm SD})}$ where σ is the attenuation coefficient [13].

The BA-DF relaying protocol is as follows. (i): S first attempts to send a packet to D along the direct S-D link. (ii): In the case where the previous attempt was not successful (because of the outage of the S-D link), S attempts to send the packet to R along the S-R link. (iii): In parallel to the potential S-D and S-R transmissions, R always attempts to send a packet to D along the R-D link. It is worthwhile noting that the transmissions from S and R can take place concurrently since the highly directive FSO links do not suffer from interference. Moreover, R operates naturally in the full-duplex mode where it can simultaneously receive and transmit packets through the photo-detector (aligned with S) and the laser (aligned with D), respectively. It is also worth noting that R can receive a packet from S only if the buffer at R is not full. Similarly, R can transmit a packet to D only if the buffer at R is not empty. Finally, and in-line with the relevant literature, S is assumed to have an infinite supply of data where a packet is generated at S in each time slot. We also assume that S is equipped with a buffer of infinite size.

III. Exact Performance Analysis

A. State Transition Matrix

A Markov chain analysis is adopted for studying the BA system [6]. A state of the Markov chain is represented by the number of packets l present in R's buffer with $0 \leq l \leq L$. We denote by $t_{l,l'}$ the probability of moving from state l to state l'. The state transition matrix $\mathbf{A}$ is defined as the $(L+1)\times(L+1)$ matrix whose (l',l)-th element is equal to $t_{l,l'}$.

When $l = 0$, transitions to the states $l' = 0$ and $l' = 1$ are possible where, in this case, the buffer is empty and no packets can be transmitted from R to D. Now, an empty buffer will remain empty because of one of the following reasons. (i): The direct S-D link is not in outage and the packet was successfully transmitted from S to D. (ii): The S-D link is in outage implying that S will attempt to send the packet to R where, in its turn, this attempt is not successful because of the outage of the S-R link. Consequently:

$$t_{0,0} = (1-p_{\rm SD}) + p_{\rm SD}p_{\rm SR} \quad ; \quad t_{0,1} = p_{\rm SD}(1-p_{\rm SR}), \quad (4)$$

where, on the other hand, the buffer size will increase by one following from the transmission of packet from S to R where this successful transmission takes place only if the S-D link is in outage while the S-R link is not in outage.

When $l = L$, transitions to the states $l' = L-1$ and $l' = L$ are possible where, in this case, the buffer is full and no packets can be transmitted from S to R. Therefore, the buffer occupancy will decrease by one (resp. remain the same) if the R-D link is not in outage (resp. is in outage). Consequently:

$$t_{L,L-1} = 1 - p_{\rm RD} \quad ; \quad t_{L,L} = p_{\rm RD}. \quad (5)$$

Consider now the case where the buffer is neither empty nor full; i.e. $l \neq 0$ and $l \neq L$. In this case, the following transitions are possible:

$$\begin{aligned} t_{l,l-1} &= (1-p_{\rm RD})\left[(1-p_{\rm SD}) + p_{\rm SD}p_{\rm SR}\right] \\ t_{l,l} &= (1-p_{\rm SD})p_{\rm RD} + p_{\rm SD}\left[p_{\rm SR}p_{\rm RD} + (1-p_{\rm SR})(1-p_{\rm RD})\right] \\ t_{l,l+1} &= p_{\rm SD}(1-p_{\rm SR})p_{\rm RD}, \end{aligned} \quad (6)$$

where the justifications are as follows. (i) For $t_{l,l-1}$, since the number of packets dropped by 1, then one packet was transmitted along R-D while no packet was transmitted along S-R. The successful transmission along the R-D link occurs when this link is not in outage with probability $1 - p_{\rm RD}$. Similarly, no packet is delivered to R in the two scenarios where either the S-D is available (with probability $1 - p_{\rm SD}$) implying that S transmitted the packet to D or the S-D link is in outage (with probability $p_{\rm SD}$) implying that S will attempt to send the packet to R without success following from the outage of the S-R link (with probability $p_{\rm SR}$). (ii) For $t_{l,l}$, the number of packets in the buffer will remain the same in the following cases. Case 1: the transmission along S-D was successful implying that no packet was transmitted from S to R. Therefore, for l to remain unchanged, no packets must exit the buffer implying that case-1 arises with probability $(1-p_{\rm SD})p_{\rm RD}$. Case 2: the S-D link is in outage. In this case,

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l will remain the same either if the S-R and R-D links are both in outage or if the S-R and R-D links are both not in outage. In fact, in the former scenario, no packet is received and no packet is transmitted while, in the latter scenario, one packet is received and one packet is transmitted implying, in both scenarios, that the number of packets present in the buffer will remain the same. (iii): For $t_{l,l+1}$, the number of packets at R increases by one implying that a packet was transmitted from S to R where this scenario arises only if the S-D link is in outage while the S-R link is not in outage. Moreover, the R-D link must be in outage with probability p_{RD}. Finally, in the case where the direct link is not activated, the transition probabilities can be obtained by replacing $p_{\text{SD}} = 1$ in (4)-(6).

B. Steady-State Distribution

Denote by π_l the probability of having l packets in the buffer at steady-state. $\pi = [\pi_0 \cdots \pi_L]^T$ is determined from [14]:

$$\mathbf{A}\pi = \pi \quad \text{s.t.} \quad \sum_{l=0}^{L} \pi_l = 1. \tag{7}$$

Equation (7) can be solved recursively as shown in Appendix A resulting in the following solution:

$$\begin{aligned}
\pi_0 &= \frac{p_{\text{RD}}(r-1)}{(r^L-1)+(r-1)\left[((1-p_{\text{SD}})+p_{\text{SD}}p_{\text{SR}})r^L-(1-p_{\text{RD}})\right]} \\
\pi_l &= \frac{r^l}{p_{\text{RD}}}\pi_0 \quad ; \quad l = 1,\dots,L-1, \\
\pi_L &= \frac{[(1-p_{\text{SD}})+p_{\text{SD}}p_{\text{SR}}]r^L}{p_{\text{RD}}}\pi_0,
\end{aligned} \tag{8}$$

where:

$$r \triangleq \frac{p_{\text{RD}}}{1-p_{\text{RD}}} \frac{p_{\text{SD}}(1-p_{\text{SR}})}{(1-p_{\text{SD}})+p_{\text{SD}}p_{\text{SR}}}. \tag{9}$$

C. Outage Probability (OP)

The system will be in outage if no packets are successfully communicated along the network's constituent links [6]:

$$P_{\text{out}} = p_{\text{SD}}\left[\pi_0 p_{\text{SR}} + \pi_L p_{\text{RD}} + (1-\pi_0-\pi_L)p_{\text{SR}}p_{\text{RD}}\right]. \tag{10}$$

The interpretation of (10) is as follows. For the system to be in outage, the direct S-D link must be in outage with probability p_{SD}. (i): When the buffer is empty buffer (with probability π_0), no packet can be transmitted along R-D implying that the outage of the S-R link will incur a system outage. (ii): When the buffer is full (with probability π_L), no packet can be transmitted from S to R implying that the outage of the R-D link will incur a system outage. (iii): When the buffer is neither full nor empty, packets can be transmitted along the S-R and R-D links implying that the outage of these two links will incur a system outage.

D. Average Packet Delay (APD)

We denote by η_{SD} and η_{SR} the effective throughputs along the S-D and S-R links, respectively:

$$\eta_{\text{SD}} = 1 - p_{\text{SD}} \quad ; \quad \eta_{\text{SR}} = p_{\text{SD}}(1-p_{\text{SR}})(1-\pi_L), \tag{11}$$

where the link S-D is available if it is not in outage. On the other hand, a packet is effectively delivered along the S-R link only if (i): the S-D link is in outage, (ii): the S-R link is not in outage and (iii): the buffer at R is not full.

The total APD can be written as: $APD = APD_{\text{S}} + APD_{\text{sys}}$ where APD_{S} stands for the delay at S (i.e. the average delay for a packet to reach the head of the queue at S) while APD_{sys} stands for the additional system delay for a packet at the head of the queue at S. Following from the analysis presented in [13], [14], $APD_{\text{S}} = \frac{1}{\eta_{\text{SD}}+\eta_{\text{SR}}} - 1$ where $\eta_{\text{SD}} + \eta_{\text{SR}}$ stands for the total output throughput from S. On the other hand, averaging the delays of the packets transmitted along the direct S-D and indirect S-R-D links results in: $APD_{\text{sys}} = \frac{\eta_{\text{SD}}}{\eta_{\text{SD}}+\eta_{\text{SR}}} \times 0 + \frac{\eta_{\text{SR}}}{\eta_{\text{SD}}+\eta_{\text{SR}}} \times \frac{\bar{L}}{\eta_{\text{SR}}}$. In fact, a fraction $\frac{\eta_{\text{SD}}}{\eta_{\text{SD}}+\eta_{\text{SR}}}$ of the total packets will be transmitted directly along the S-D link and will not experience any additional delay. On the other hand, the remaining fraction of packets will experience an additional queuing delay at R where this delay can be calculated from $\frac{\bar{L}}{\eta_{\text{SR}}}$ following from Little's law [15] where $\bar{L} = \sum_{l=0}^{L} l\pi_l$ stands for the average queue length. Combining the above expressions results in:

$$APD = \frac{\bar{L}+1}{\eta_{\text{SD}}+\eta_{\text{SR}}} - 1. \tag{12}$$

IV. Asymptotic Analysis

A. Methodology

We next provide an asymptotic analysis that holds for large values of P_M resulting in $p_{\text{SD}} \ll 1$, $p_{\text{SR}} \ll 1$ and $p_{\text{RD}} \ll 1$. We assume in what follows that $L > 1$ since, in the case $L = 1$, the buffer is either empty or full all of the time. This assumption is consistent with the related literature where often large buffer sizes are assumed. As in [14], the asymptotic analysis will revolve around the identification of a closed-subset of states $\mathcal{S}$ where the transition probability of moving from any state inside $\mathcal{S}$ to any state outside $\mathcal{S}$ tends to zero [16]. Therefore, at steady-state, $\sum_{l\in\mathcal{S}} \pi_l \to 1$ while $\pi_l \to 0$; $\forall\, l \notin \mathcal{S}$. In fact, after a certain number of transitions among the transient states outside $\mathcal{S}$, the Markov chain will eventually move to $\mathcal{S}$ and remain in this closed-subset since the transition probabilities out of this subset tend to zero.

B. Steady-State Distribution

In Appendix B, we prove that $\mathcal{S} = \{0,1\}$ in the presence of a direct link. On the other hand, when the direct link is not available, the closed subsets are given by $\mathcal{S} = \{0,1\}$ for $p_{\text{RD}} < p_{\text{SR}}$ (R closer to D) and $\mathcal{S} = \{L-1, L\}$ for $p_{\text{SR}} < p_{\text{RD}}$ (R closer to S). We also prove that the corresponding non-zero steady-state probabilities tend to the following values:

$$\begin{cases}
(\pi_0,\pi_1) \to \left(\frac{1-p_{\text{SD}}-p_{\text{RD}}}{1-p_{\text{RD}}}, \frac{p_{\text{SD}}}{1-p_{\text{RD}}}\right), & p_{\text{SD}} \neq 1; \\
(\pi_0,\pi_1) \to (p_{\text{SR}}, 1-p_{\text{SR}}), & p_{\text{SD}} = 1 \ \&\ p_{\text{RD}} < p_{\text{SR}}; \\
(\pi_{L-1},\pi_L) \to (1-p_{\text{RD}}, p_{\text{RD}}), & p_{\text{SD}} = 1 \ \&\ p_{\text{SR}} < p_{\text{RD}}.
\end{cases} \tag{13}$$

From (13), it can be observed that activating the direct link has the impact of making the buffer empty most of the time at steady-state where this observation holds irrespective of the

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position of R relative to S and D. This observation holds since $\pi_0 \rightarrow \frac{1-p_{\rm SD}-p_{\rm RD}}{1-p_{\rm RD}} \approx 1$. On the other hand, when the direct link is not available, the buffer tends to have one packet most of the time ($\pi_1 \rightarrow 1-p_{\rm SR} \approx 1$) when R is closer to D while it tends to have $L-1$ packets ($\pi_{L-1} \rightarrow 1-p_{\rm RD} \approx 1$) when R is closer to S. In fact, in the latter case, when $d_{\rm RD} < d_{\rm SR}$, the relatively poor quality of the longer hop S-R reduces the occupancy of the buffer since, on average, at R retransmissions are more successful than the reception. On the other hand, when $d_{\rm SR} < d_{\rm RD}$, the R-D link constitutes the bottleneck link where at R the reception is more frequent than the retransmissions with the direct impact of increasing the number of packets present in the buffer. In this context, the presence of the direct link leverages the occupancy of the buffer at R since S will attempt to send a packet to R only when the direct link S-D is in outage thus significantly reducing the arrival packet rate at R.

C. Outage Probability (OP) and Diversity Order

Replacing (13) in (10) results in the following asymptotic expression of the system OP:

$$P_{\rm out} \rightarrow \begin{cases} P_{\rm out,BA}^{\rm (direct)} \triangleq p_{\rm SD}p_{\rm SR}, & p_{\rm SD} \neq 1; \\ P_{\rm out,BA}^{\rm (no)} \triangleq (\max\{p_{\rm SR}, p_{\rm RD}\})^2, & p_{\rm SD} = 1. \end{cases} \quad (14)$$

Equation (14) shows that in the presence of a direct link, OP does not depend on the outage probability along the R-D link. This is justified by the fact that the buffer is empty most of the time implying that no transmissions will take place along the R-D link. Now, when the direct link is not available, OP is dominated by the weakest of the two hops S-R and R-D.

The OP in (14) must be compared with that of a buffer-free (BF) system where $P_{\rm out} = p_{\rm SD}[1-(1-p_{\rm SR})(1-p_{\rm RD})] \approx p_{\rm SD}[p_{\rm SR}+p_{\rm RD}]$ [5]. This OP can be written as:

$$P_{\rm out} \rightarrow \begin{cases} P_{\rm out,BF}^{\rm (direct)} \triangleq p_{\rm SD}\max\{p_{\rm SR}, p_{\rm RD}\}, & p_{\rm SD} \neq 1; \\ P_{\rm out,BF}^{\rm (no)} \triangleq \max\{p_{\rm SR}, p_{\rm RD}\}, & p_{\rm SD} = 1 \end{cases} \quad (15)$$

For $P_M \gg 1$, the probabilities in (1), (2) and (3) behave asymptotically as $p_{\rm SD} \rightarrow P_M^{-\beta_{\rm SD}}$, $p_{\rm SR} \rightarrow P_M^{-\beta_{\rm SR}}$ and $p_{\rm RD} \rightarrow P_M^{-\beta_{\rm RD}}$, respectively [14]. This implies that the diversity orders along the S-D, S-R and R-D links are equal to $\beta_{\rm SD}$, $\beta_{\rm SR}$ and $\beta_{\rm RD}$, respectively. Consequently, from (14) and (15), the diversity orders achieved by the BA and BF systems are:

$$\begin{cases} \delta_{\rm out,BA}^{\rm (direct)} = \beta_{\rm SD}+\beta_{\rm SR}, & p_{\rm SD} \neq 1; \\ \delta_{\rm out,BA}^{\rm (no)} = 2\min\{\beta_{\rm SR}, \beta_{\rm RD}\}, & p_{\rm SD} = 1. \end{cases} \quad (16)$$

$$\begin{cases} \delta_{\rm out,BF}^{\rm (direct)} = \beta_{\rm SD}+\min\{\beta_{\rm SR}, \beta_{\rm RD}\}, & p_{\rm SD} \neq 1; \\ \delta_{\rm out,BF}^{\rm (no)} = \min\{\beta_{\rm SR}, \beta_{\rm RD}\}, & p_{\rm SD} = 1. \end{cases} \quad (17)$$

In what follows, the impact of the direct link on the achievable diversity orders will be highlighted where the two following cases arise depending on the position of R.

Case 1: R is closer to D resulting in $\beta_{\rm SR} < \beta_{\rm RD}$. In this case, (16) and (17) imply that $\delta_{\rm out,BA}^{\rm (direct)} = \beta_{\rm SD}+\beta_{\rm SR}$, $\delta_{\rm out,BA}^{\rm (no)} = 2\beta_{\rm SR}$, $\delta_{\rm out,BF}^{\rm (direct)} = \beta_{\rm SD}+\beta_{\rm SR}$ and $\delta_{\rm out,BF}^{\rm (no)} = \beta_{\rm SR}$. Consequently:

$$\delta_{\rm out,BF}^{\rm (no)} < \delta_{\rm out,BF}^{\rm (direct)} = \delta_{\rm out,BA}^{\rm (direct)} < \delta_{\rm out,BA}^{\rm (no)}, \quad (18)$$

where the last inequality follows from $d_{\rm SR} < d_{\rm SD} \Rightarrow \beta_{\rm SD} < \beta_{\rm SR}$.

Equation (18) implies that, in the presence of a direct link, the BA system achieves the same diversity order as the BF system. Moreover, from an outage probability point of view, it is better not to activate the direct link in this case (R is closer to D). This is justified by the fact that whether the direct link is activated ($\pi_0 \rightarrow 1$) or not ($\pi_1 \rightarrow 1$ for $d_{\rm RD} < d_{\rm SR}$), the buffer occupancy is very low in both scenarios implying that the activation of the direct link does not effectively contribute to reducing the packet arrival rate at R.

Case 2: R is closer to S ($d_{\rm SR} < d_{\rm RD}$) resulting in $\beta_{\rm RD} < \beta_{\rm SR}$. In this case, (16) and (17) imply that $\delta_{\rm out,BA}^{\rm (direct)} = \beta_{\rm SD}+\beta_{\rm SR}$, $\delta_{\rm out,BA}^{\rm (no)} = 2\beta_{\rm RD}$, $\delta_{\rm out,BF}^{\rm (direct)} = \beta_{\rm SD}+\beta_{\rm RD}$ and $\delta_{\rm out,BF}^{\rm (no)} = \beta_{\rm RD}$. Consequently, the following inequalities follow:

$$\delta_{\rm out,BF}^{\rm (no)} < \delta_{\rm out,BF}^{\rm (direct)} < \delta_{\rm out,BA}^{\rm (no)}, \quad (19)$$

$$\delta_{\rm out,BF}^{\rm (no)} < \delta_{\rm out,BF}^{\rm (direct)} < \delta_{\rm out,BA}^{\rm (direct)}, \quad (20)$$

where the second inequality in (19) follows since $\beta_{\rm SD} < \beta_{\rm RD}$ (since $d_{\rm RD} < d_{\rm SD}$ in general) while the second inequality in (20) follows since $\beta_{\rm RD} < \beta_{\rm SR}$.

Equations (19)-(20) show that, in the considered case, BA systems profit from an enhanced diversity order compared to BF systems. On the other hand, the comparison between $\delta_{\rm out,BA}^{\rm (no)}$ and $\delta_{\rm out,BA}^{\rm (direct)}$ depends on the specific values of the link distances. In general, the parameter β of the gamma-gamma distribution decreases very rapidly with the link distance implying that $\beta_{\rm SR}$ associated with the link S-R (that is the shortest among the links S-D, S-R and R-D) is much bigger than $\beta_{\rm SD}$ and $\beta_{\rm RD}$ associated with the other two longer hops. Therefore, for practical network setups:

$$\delta_{\rm out,BA}^{\rm (no)} < \delta_{\rm out,BA}^{\rm (direct)}, \quad (21)$$

where this inequality was validated numerically for a wide range of the link distances satisfying $d_{\rm SR} < d_{\rm RD} < d_{\rm SD}$.

Equation (21) shows that, when R is closer to S, activating the direct link is advantageous from a diversity order perspective. In this case, activating the direct link reduces the packet congestion at the relay from a buffer that is almost full ($\pi_{L-1} \rightarrow 1$ for $d_{\rm SR} < d_{\rm RD}$) for a buffer that is empty most of the time ($\pi_0 \rightarrow 1$).

D. Average Packet Delay (APD)

In the presence of a direct link, the quantities in (11) tend to $\eta_{\rm SD} \rightarrow 1$ and $\eta_{\rm SR} \rightarrow 0$ implying that $APD \rightarrow \bar{L}$ following from (12). In this case, $\pi_0 \rightarrow 1 \Rightarrow \bar{L} \rightarrow 0$ implying that $APD \rightarrow 0$. In the absence of a direct link, $p_{\rm SD} = 1$ implying that $\eta_{\rm SD} = 0$ and $\eta_{\rm SR} \rightarrow 1$ from (11). Consequently, (12) results in $APD \rightarrow \bar{L}$ where $\bar{L} = 1$ when $p_{\rm RD} < p_{\rm SR}$ and $\bar{L} = L-1$ when $p_{\rm SR} < p_{\rm RD}$. Therefore:

$$APD \rightarrow \begin{cases} 0, & p_{\rm SD} \neq 1; \\ 1, & p_{\rm SD} = 1 \;\&\; p_{\rm RD} < p_{\rm SR}; \\ L-1, & p_{\rm SD} = 1 \;\&\; p_{\rm SR} < p_{\rm RD}. \end{cases} \quad (22)$$

Equation (22) shows that activating the direct link is always useful in decreasing the delay. This delay approaches zero asymptotically showing that, in this case, the BA system profits from the zero-delay that is inherent to BF systems.

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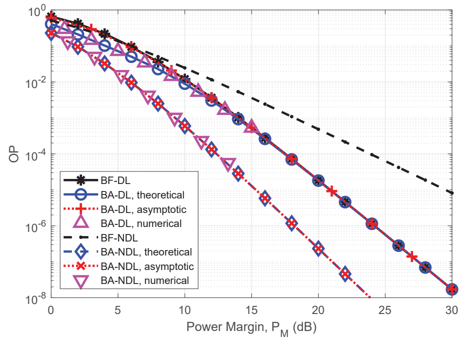

Fig. 1. OP for $(d_{\mathrm{SR}}, d_{\mathrm{RD}}) = (2.5, 1.5)$ km with direct link (DL) and with no direct link (NDL).

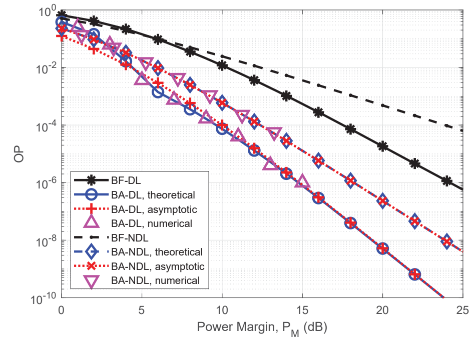

Fig. 3. OP for $(d_{\mathrm{SR}}, d_{\mathrm{RD}}) = (1.5, 2.5)$ km with direct link (DL) and with no direct link (NDL).

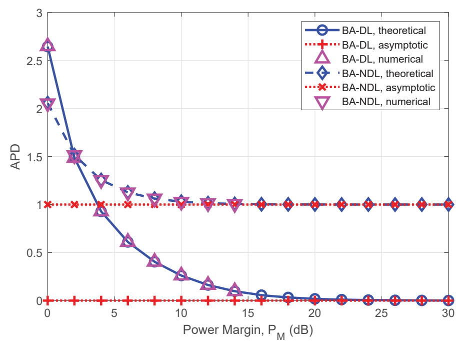

Fig. 2. APD for $(d_{\mathrm{SR}}, d_{\mathrm{RD}}) = (2.5, 1.5)$ km with direct link (DL) and with no direct link (NDL).

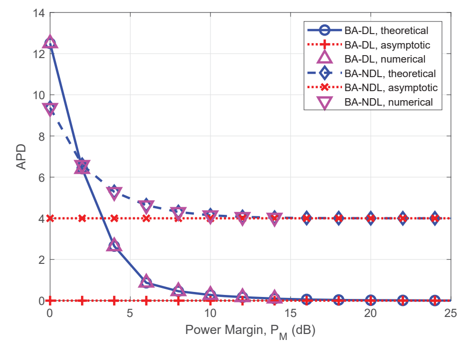

Fig. 4. APD for $(d_{\mathrm{SR}}, d_{\mathrm{RD}}) = (1.5, 2.5)$ km with direct link (DL) and with no direct link (NDL).

As a conclusion, when R is closer to D, activating the direct link minimizes the delay at the expense of reducing the diversity order following from (18) and (22). On the other hand, when R is closer to S, activating the direct link is very appealing since this activation jointly decreases the delay and increases the diversity order following from (21) and (22).

V. Numerical Results

We next present some numerical results that support the findings reported in the previous sections. The refractive index structure parameter and the attenuation constant are set to $C_n^2 = 1.7 \times 10^{-14}$ m$^{-2/3}$ and $\sigma = 0.44$ dB/km, respectively. The distance between S and D is fixed to $d_{\mathrm{SD}} = 4$ km. We also fix the buffer size to $L = 5$.

Fig. 1 and Fig. 2 show the OP and APD in the scenario where R is placed closer to D with $d_{\mathrm{SR}} = 2.5$ km and $d_{\mathrm{RD}} = 1.5$ km. Results in Fig. 1 and Fig. 2 show the close match between the theoretical results and numerical results that were obtained by a discrete event simulator. Results also highlight on the accuracy of the asymptotic OP and APD expressions in (14) and (22) for predicting the system performance for large values of P_M. As predicted by (18), activating the direct link reduces the diversity order from $\delta_{\text{out,BA}}^{(\text{no})} = 3.57$ to $\delta_{\text{out,BA}}^{(\text{direct})} = 3.07$. In this case, in coherence with (18), Fig. 1 shows that the BA and BF systems show practically the same OP performance when the direct link is activated while the best performance is achieved by the BA system with no direct link. Finally, as highlighted in (22), activating the direct link reduces the asymptotic APD from 1 to 0.

Fig. 3 and Fig. 4 show the OP and APD in the scenario where R is placed closer to R with $d_{\mathrm{SR}} = 1.5$ km and $d_{\mathrm{RD}} = 2.5$ km. As predicted by (21) and (22), activating the direct link jointly reduces the OP and APD highlighting that the BA system with direct link constitutes the best solution in this operating scenario. In this case, activating the direct link enhances the diversity order from $\delta_{\text{out,BA}}^{(\text{no})} = 3.57$ to $\delta_{\text{out,BA}}^{(\text{direct})} = 4.58$ while reducing the asymptotic APD from 4 to 0. Finally, results in Fig. 3 clearly highlight the superiority of BA-relaying compared to BF-relaying.

VI. Conclusion

We derived the outage probability, average packet delay and diversity gains of a three-node FSO system in the case where the relay is equipped with a finite-size buffer. The derived simple asymptotic expressions of the targeted performance measures helped in shedding more light on the system performance and in delineating the advantages and disadvantages that result from activating the direct link.

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Appendix A

The first equation in (7), resulting from the first row of $\mathbf{A}$, can be written as $t_{0,0}\pi_0 + t_{l,l-1}\pi_1 = \pi_0$. Replacing $t_{0,0}$ and $t_{l,l-1}$ by their values from (4) and (6), respectively, results in:

$$\frac{\pi_1}{\pi_0} = \frac{r}{p_{\text{RD}}}, \tag{23}$$

where r is defined in (9). The second equation in (7) is given by $t_{0,1}\pi_0 + t_{l,l}\pi_1 + t_{l,l-1}\pi_2 = \pi_1$. Dividing both sides of this equation by π_1 while invoking (23) results in $\frac{\pi_2}{\pi_1} = \frac{t_{l,l+1}}{t_{l,l-1}} = r$. Similarly, equation $l'+1$ for $l' = 2, \dots, L-2$ can be written as $t_{l,l+1}\pi_{l'-1} + t_{l,l}\pi_{l'} + t_{l,l-1}\pi_{l'+1} = \pi_{l'}$. Dividing both sides of this equation by $\pi_{l'}$ results in $t_{l,l+1}\frac{\pi_{l'-1}}{\pi_{l'}} + t_{l,l-1}\frac{\pi_{l'+1}}{\pi_{l'}} = 1 - t_{l,l}$. Replacing recursively $\frac{\pi_{l'-1}}{\pi_{l'}} = \frac{1}{r}$ for $l' = 2, \dots, L-2$ results in $\frac{\pi_{l'+1}}{\pi_{l'}} = r$ implying the following set of relations:

$$\frac{\pi_2}{\pi_1} = \frac{\pi_3}{\pi_2} = \cdots = \frac{\pi_{L-1}}{\pi_{L-2}} = r. \tag{24}$$

The L-th equation in (7) is given by $t_{l,l+1}\pi_{L-2} + t_{l,l}\pi_{L-1} + t_{L,L-1}\pi_L = \pi_{L-1}$ where $t_{L,L-1}$ is given in (5). Dividing by π_{L-1} while invoking (24) results in:

$$\frac{\pi_L}{\pi_{L-1}} = [(1 - p_{\text{SD}}) + p_{\text{SD}}p_{\text{SR}}]r. \tag{25}$$

Equations (23), (24) and (25) result in $\pi_1 = \frac{r}{p_{\text{RD}}}\pi_0, \pi_2 = \frac{r^2}{p_{\text{RD}}}\pi_0, \dots, \pi_{L-1} = \frac{r^{L-1}}{p_{\text{RD}}}\pi_0, \pi_L = \frac{[(1-p_{\text{SD}})+p_{\text{SD}}p_{\text{SR}}]r^L}{p_{\text{RD}}}\pi_0$ which correspond to the second and third relations in (8). Finally, replacing these values in the equation $\sum_{l=0}^{L}\pi_l = 1$ and solving for π_0 results in the first relation in (8).

Appendix B

Consider first the case where the direct link is available. From (4), $t_{0,0} \to 1 - p_{\text{SD}}$ and $t_{0,1} \to p_{\text{SD}}$ for $p_{\text{SD}} \ll 1$, $p_{\text{SR}} \ll 1$ and $p_{\text{RD}} \ll 1$. Similarly, from (6), $t_{l,l-1} \to (1-p_{\text{RD}})(1-p_{\text{SD}}) \approx 1 - p_{\text{SD}} - p_{\text{RD}}$, $t_{l,l} \to p_{\text{SD}} + p_{\text{RD}}$ and $t_{l,l+1} \to 0$. Therefore, the subset $\mathcal{S} = \{0, 1\}$ is closed since the transition probabilities that do not tend to zero correspond to $\{t_{0,0}, t_{0,1}, t_{1,0}, t_{1,1}\}$ (while $t_{1,2} \to 0$). In other words, starting from the state $l = 0$ or $l = 1$, transitions are possible only to the states $l' = 0$ and $l' = 1$ highlighting that these two states define a closed subset. Now, solving the steady-state equation $(1-p_{\text{SD}})\pi_0 + (1 - p_{\text{SD}} - p_{\text{RD}})\pi_1 = \pi_0$ subject to $\pi_0 + \pi_1 = 1$ results in the solution given in the first line of (13).

Consider now the case where the direct link is not activated. Replacing p_{SD} by 1 in (4) and (6) results in:

$$t_{0,0} = p_{\text{SR}} \;\;;\;\; t_{0,1} = 1 - p_{\text{SR}} \tag{26}$$

$$t_{l,l-1} = p_{\text{SR}}(1-p_{\text{RD}}) \;\;;\;\; t_{l,l} = (1-p_{\text{SR}})(1-p_{\text{RD}}) + p_{\text{SR}}p_{\text{RD}}$$
$$t_{l,l+1} = (1-p_{\text{SR}})p_{\text{RD}}. \tag{27}$$

In this case, the following scenarios arise. (i): R is closer to D resulting in $p_{\text{RD}} < p_{\text{SR}}$. From (27), $t_{l,l-1} \to p_{\text{SR}}$, $t_{l,l} \to 1 - p_{\text{SR}}$ and $t_{l,l+1} \to 0$. This implies that the set $\mathcal{S} = \{0, 1\}$ is closed with the following transition probabilities $t_{0,0} = t_{1,0} = p_{\text{SR}}$ and $t_{0,1} = t_{1,1} = 1 - p_{\text{SR}}$. Now, solving the equation $t_{0,0}\pi_0 + t_{1,0}\pi_1 = \pi_0$ subject to $\pi_0 + \pi_1 = 1$ results in the solution given in the second line of (13). (ii): R is closer to S resulting in $p_{\text{SR}} < p_{\text{RD}}$. From (27), $t_{l,l-1} \to 0$, $t_{l,l} \to 1 - p_{\text{RD}}$ and $t_{l,l+1} \to p_{\text{RD}}$. Combining this result with (5) implies that the set $\mathcal{S} = \{L-1, L\}$ is closed with the following transition probabilities $t_{L-1,L} = t_{L,L} = p_{\text{RD}}$ and $t_{L-1,L-1} = t_{L,L-1} = 1 - p_{\text{RD}}$. Now, solving the equation $t_{L-1,L-1}\pi_{L-1} + t_{L,L-1}\pi_L = \pi_{L-1}$ subject to $\pi_{L-1} + \pi_L = 1$ results in the solution given in the third line of (13).

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